Concept of continuum → Continuous medium

Properties like pressure, temp. density etc are defined continuous function of space. It assumes that molecules are closely spaced ie. Gap between molecules is almost zero. But this is not true gases at very low pressure;

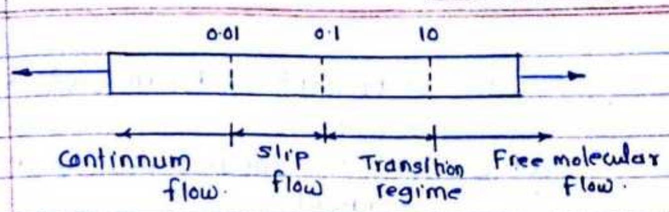
By Avogadro hypothesis.
6.023×10^{23} molecules per 22.4 l.

Density 2.7×10^{25} molecules/m^3 at normal temp. & pressure.

Decrease in pressure results in increase in gap ie. mean free path of molecules & also decrease in cohesive force, hence concept of continnum doesn't valid.

λ → Mean free path
L → Characterstics dimension of the problem ie. for pipe dia. d.
$\frac{\lambda}{L}$ → Knudsen number serves criteria to define continnum.

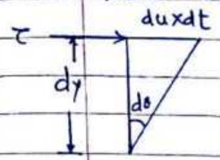
$\frac{\lambda}{L} < 0.01$ (Continnum is valid)



Viscosity → Viscosity is the defined as the property of a fluid which offer resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

$$\tau \propto \frac{d\theta}{dt}$$

Shear stress is directly proportional to rate of change of angle w.r.t. time



$d = s \times t = du \times dt$

$\tan d\theta = \frac{du\,dt}{dy}$

If θ is small $\tan\theta = \theta$

ie. $d\theta = \frac{du \times dt}{dy}$

$\therefore \frac{d\theta}{dt} = \frac{du}{dy}$

$$\tau \propto \frac{du}{dy}$$

$$\boxed{\tau = \mu \frac{du}{dy}}$$ Newtons Law of viscosity.

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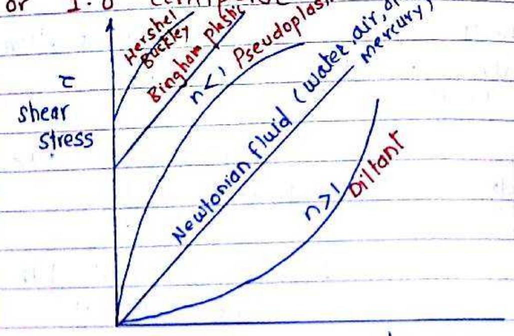
μ = Coefficient of dynamic viscosity or viscosity.

$\frac{du}{dy}$ = Rate of shear strain or velocity gradient.

SI unit → $\frac{N \cdot S}{m^2}$ or Pa·Sec. MKS - $\frac{kg}{sec}$

* 1 Poise = $\frac{1}{10} \frac{N \cdot S}{m^2}$

* Viscosity of water at 20°C is 0.01 Poise or 1.0 centipoise.



Newtonian fluid → Shear stress is directly proportional to the rate of shear strain.

Non-newtonian fluid → τ is not proportional $\frac{du}{dy}$. It follows power law model.

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$$\tau = m\left(\frac{du}{dy}\right)^n$$

$$= m\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy}$$

This model also known as Ostwald-de-wadele model.

$$\frac{\tau}{du/dy} = \mu = m\left|\frac{du}{dy}\right|^{n-1} = \text{Apperant viscosity.}$$

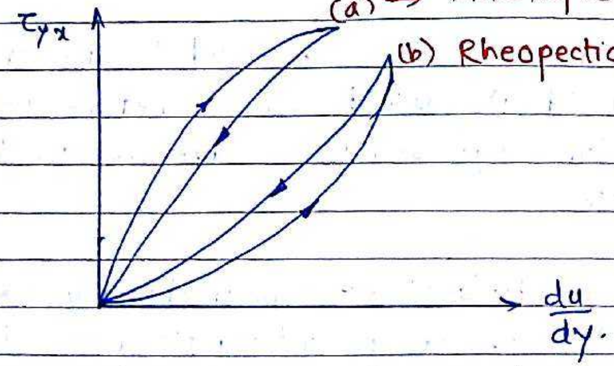
where m = Flow behaviour index.
n = Flow Consistency index.

Bingham plastic → Fluid is one that requires a finite yield stress before beginning to flow. eg. Sewage Sludge, mud, clay, toothpaste

Pseudoplastic → Shear thinning → Appearant viscosity of pseudoplastic fluid decreases with increase in shear rate may be qualitatively attributed to breakdown of loosely bonded aggregates by shearing effect of flow.
eg. aqueous or non aqueous suspensions of of polymers.

~~Bing~~ Hershel-Buckley → It has initial yield stress to flow but it becomes pseudoplastic afterwards.

Dildant → Increment in appearant viscosity with increase in the shear rate for dilatant fluids may be due to the shift of a closely packed perticulate system to a more open arrangement under shear which may entrap some of the liquid.
eg. Aqueous suspensions of magnetite, galena & ferro silicons.



* Appearant viscosity of thixotropic fluids decreases with time under constant shear.
eg. Water suspension in bentonitic clay drilling fluid used in petroleum industry.

* Apperent viscosity increases with increase in time for a given shear rate is called Rheopectic fluids eg. Gypsum pastes, Printers inks.

Propeeties of Fluids →

* Density or mass density → Ratio of mass of fluid to its volume. SI unit kg/m^3

Value of water density is $1\ gm/cm^3$ OR $1000\ kg/m^3$ at $4^\circ C$.

* Specific Weight or Weight density → Ratio of weight of fluid to its volume.

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{mg}{v}.$$

$$w = \rho g \quad N/m^3$$

For water $= w = 9.81 \times 1000 = 9810\ N/m^3$

* Specific Volume → Ratio of volume of fluid to its mass. Reciprocal of density.

$$\upsilon = \frac{1}{\rho} = \frac{v}{m} = m^3/kg.$$

* Specific gravity or relative density → Ratio of weight density or density of a fluid to the weight density or density of standard fluid.

For liquids water is standard fluid.

for Gases air is standard fluid

For mecury $S = 13.6$ ie. its density $13.6 \times 1000 = 13600 \ kg/m^3$.

* **Kinematic Viscosity** → ν → Ratio of viscosity to the density of fluid.

$$\nu = \frac{\mu}{\rho} \quad \frac{m^2}{sec}.$$

It's kinematic quantity as doesn't contain any unit of mass. It represent relative ability of fluid to diffuse a disturbance in momentum as compared to its ability of sustaining the original momentum.

C.G.S unit $\frac{cm^2}{sec}$ ie. stokes.

1 stoke → $10^{-4} \ m^2/sec$.

* Compressibility → (β)

Compressibility of any substance is the measure of its change in volume under the action of external forces namely normal compressive forces.

$$\boxed{\beta = \frac{-dV/V}{dp}}$$

... −ve sign decrease in volume

* Compressibility usually defined for gases.
* β is higher means easy to compress ie. air.

$$\rho = \frac{m}{V} \qquad m = \rho V = \text{constant}.$$

differentiating $d\rho \cdot V + \rho \, dV = 0$

$$\frac{d\rho}{\rho} = -\frac{dV}{V}$$

$$\boxed{\beta = \frac{d\rho/\rho}{dp}}$$

* **Bulk modulus** → Degree of compressibility of a substance is characterized by the bulk modulus / Reciprocal of compressibility.

$$E = \frac{1}{\beta} = \frac{dp}{-dV/V} = \frac{-V\,dp}{dV}$$

* Bulk modulus is usually defined for liquids.
* Bulk modulus is higher means difficult to compress ie. incompressible.

E for water → $2 \times 10^6 \ kN/m^2$

air → $100 \ kN/m^2$

It indicates air is 20,000 times more compressible than liquid water.

* Variation of viscosity with temp →

Liquids → Cohesive forces + molecular momentum transfer

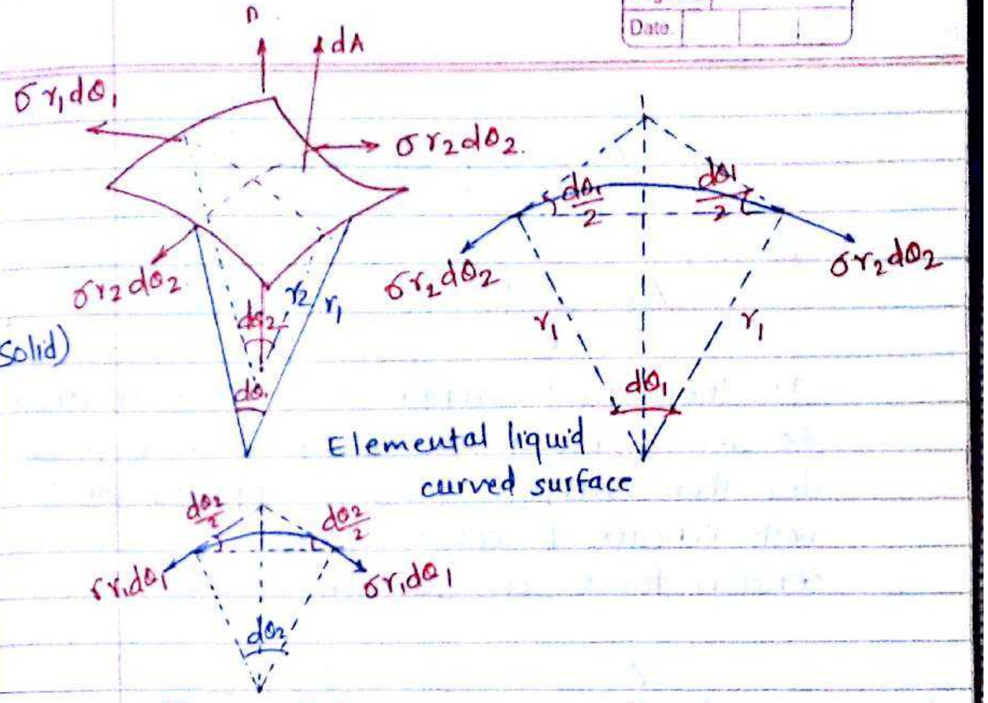
In liquids Temp ↑ Viscosity ↓ due to decrease in cohesive forces.

But in gases cohesive forces are very small & molecular momentum predominates hence increase in viscosity

* Surface Tension → Consider interface between two fluid phases (liquid & ~~Gas~~ Solid) molecules in liquid are attracted towards the interface by vander waals forces. However molecules doesnot stick to the wall because of Brownian motion. For liquid Gas interface liquid particles at interface are attracted by liquid particles which forms pull. If the interfacial molecules do not possess sufficient energy to overcome this net attraction & remain at the interface, they would ultimately dissolved in the liquid & get lost. However when an interface is formed this does not occur. This implies that interface has sufficient energy to overcome any net driving force on its molecules & allow them to be located on the interface. This energy is known as interfacial energy or surface energy.

Surface energy per unit area is known as surface tension

$$\sigma = \frac{\text{Energy}}{\text{Area}} = \frac{J}{m^2} = \frac{N\cdot m}{m^2} = \frac{N}{m}.$$



Force Balance in the direction perpendicular to the surface finish.

Let the surface be subjected to uniform pressure P_i & P_o at its concave & convex sides

$$\cancel{2}\sigma r_2 d\theta_2 \sin\left(\frac{d\theta_1}{\cancel{2}}\right) + \cancel{2}\sigma r_1 d\theta_1 \sin\left(\frac{d\theta_2}{\cancel{2}}\right) = (P_i - P_o)\, r_1 r_2\, d\theta_1 d\theta_2$$

For small angles.

$$\sin\frac{d\theta_1}{2} \approx \frac{d\theta_1}{2} \qquad \sin\frac{d\theta_2}{2} \approx \frac{d\theta_2}{2}$$

$$(\sigma r_2 + \sigma r_1)\, \cancel{d\theta_1 d\theta_2} = (P_i - P_o)\, r_1 r_2\, \cancel{d\theta_1 d\theta_2}$$

$$\Delta P = \frac{\sigma}{r_1} + \frac{\sigma}{r_2} \qquad (1)$$

∴ Young Laplace eq^n.

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$$\Delta P = \sigma\left[\frac{1}{r_1}+\frac{1}{r_2}\right]$$

where,

$\Delta P = P_i - P_o$

└ Excess pressure

If the liquid surface co-exists with another immiscible fluid or gas. on both sides then surface tension appears on the both concave & convex sides. Net surface tension force on surface will be twice.

$$\Delta P = 2\left(\frac{\sigma}{r_1}+\frac{\sigma}{r_2}\right) \quad -②$$

Special Cases →

i) For Spherical liquid drop $= r_1 = r_2 = r$

eq^n ① becomes

$$\Delta P = \frac{2\sigma}{r}$$

ii) For liquid jet Cylindrical $r_2 = r \quad r_1 = \infty$

$$\Delta P = \frac{\sigma}{r}$$

iii) For spherical bubble : $r_1 = r_2 = r$ & pressure from two sides, eq^n ② becomes

$$\Delta P = \frac{2\times 2\sigma}{r} = \frac{4\sigma}{r}$$

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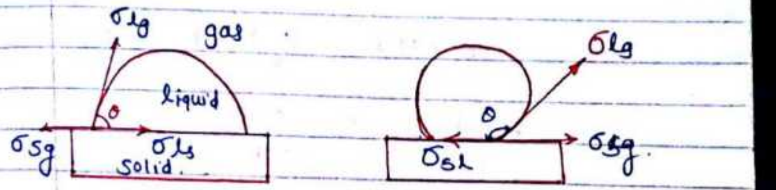
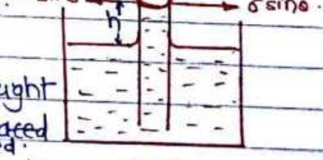


Water glass	Mercury glass
Hydrophilic	Hydrophobic
$\theta \rightarrow$ Contact angle	$\theta \rightarrow$ Contact angle
$\theta \rightarrow 0 \le \theta \le 90°$	$\theta > 90°$
wetting	Non-wetting.

Imp
* Fluid always try to contact. Water droplet becomes sphere because water droplet trying to achieve min. surface area & ie. min surface energy. Hence from all shapes sphere has least surface area.

* Capillarity → Phenomenon of rise or fall of a liquid surface in a small tube relative to the adjcent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression.



Upward force = Weight of water ~~displaced~~ lifted.

$\frac{\text{Surface tension}}{\text{Length}} \times \text{length} = \rho V g$.

→ ce exerted by water on the turbine blades decreases. Hence the workdone by water or O/P power becomes less hence thus η decreases.

$$\sigma \cos\theta \times \pi d = \frac{\pi}{4} d^2 \times h \times \rho g$$

$$\boxed{h = \frac{4\sigma\cos\theta}{\rho g d}}$$

* θ between water and clean glass tube is approx. equal to zero. $h = \frac{4\sigma}{\rho g d}$

* θ for mercury & glass tube $128°$

* For petrol surface tension is very less hence droplet not formed by petrol.

* $h \propto \frac{1}{d}$ ie. lesser the diameter more will water lifts. hence in trees water lifts from roots to the leafs.

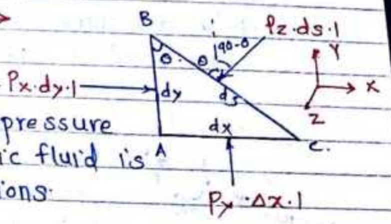
Vapour Pressure & Cavitation →

When water heated at atmospheric pressure water boils at $100°c$. When vapourization takes place the molecules escapes from the free surface of the liquid. If pressure of water at temp $20°c$ reduced by some means such that pressure equal or less than vapour pressure the water starts boiling at $20°c$. Thus a liquid may boil even at ordinary temp.

* Effects of cavitation →
1) The metallic surfaces are damaged & cavities are formed on the surfaces.
2) The η_{Turb} decreases due to cavitation
3) Noise & vibrations
4) Due to pitting action the surface of the turbine blade becomes rough & the for-

If pressure is reduced to vapour pressure at any point in flowing system, the vaporization of the liquid starts. The bubbles of this vapour carried by the flowing liquid into the region of high pressure where they collapse giving to rise to high impact pressure. The pressure developed by the collapsing bubbles is so high that the material from the adjoining boundaries gets eroded. The metallic surfaces above which the liquid is flowing is subjected to these high pressures which cause pitting action on the surface. This phenomenon is known as cavitation.

Pascal's Law →



It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.

Forces acting on fluid element
1) Pressure forces normal to the surfaces.
2) Weight of the element in the vertical direction. [$\rho \times V \times g$]

Force on AB → $P_x \times$ Area of AB
→ $P_x \cdot dy \cdot 1$

force on BC → $P_z \times ds \times 1$

force on AC → $P_y \times dx \times 1$

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Fluid is at rest, hence.

$$\Sigma F_x = 0$$

$$P_x \times dy \cdot 1 - P \cdot ds \cdot 1 \times \sin(90-\theta) = 0$$

$$P_x \times dy \cdot 1 - P_z \, ds \times 1 \cos\theta = 0$$

$$ds \cos\theta = AB = dy$$

$$P_x \, dy \cdot 1 = P_z \, dy \cdot 1 \qquad P_x = P_z.$$

$$\Sigma F_y = 0$$

$$P_y \times dx \times 1 - P_z \times ds \times 1 \cos(90-\theta) - \frac{dx\,dy}{2} \times 1 \times \rho \times g = 0$$

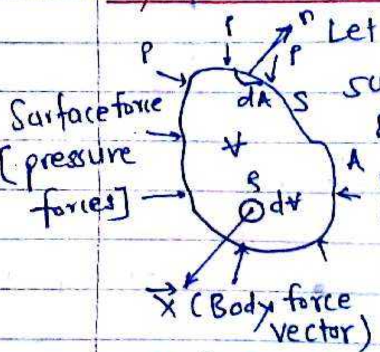
Since element is small weight of element is negligible.

$$P_y \, dx = P_z \, dx \qquad P_y = P_z$$

Hence $\boxed{P_x = P_y = P_z}$ or $\sigma_x = \sigma_y = \sigma_z$ Comp. stress.

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.

Hydrostatic Pressure →



Let consider fluid element of surface area A & volume $\forall$ & in which element $d\forall$ & having density ρ.

$\vec{X} \Rightarrow$ Body force vector per unit mass.

Total Body force $\vec{F_B} = \iiint_{\forall} \vec{X} \rho \, d\forall$

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Total Surface force $= \iint_A -\vec{n} \, P \, dA$

Body under equilibrium,

$$\vec{F_B} + \vec{F_S} = 0$$

$$\iiint_{\forall} \bar{X} \rho \, d\forall + \iint_A -\vec{n} \, P \, dA = 0$$

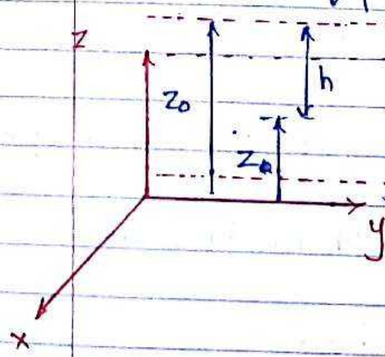
Gauss Divergence theorem, change surface integral to volume integral by gradient ∇.

$$\iiint \rho \bar{X} \, d\forall + \iiint -\nabla P \, d\forall = 0$$

$$\iiint_{\forall} (\rho \vec{X} - \nabla P) \, d\forall = 0$$

It is valid for any volume of fluid & hence

$$\nabla P - \rho \bar{X} = 0 \qquad \boxed{\nabla P = \rho \vec{X}}$$



$$\nabla P = \hat{i}\frac{\partial P}{\partial x} + \hat{j}\frac{\partial P}{\partial y} + \hat{k}\frac{\partial P}{\partial z}$$

$$\vec{X} = \hat{i}\rho X_x + \hat{j}\rho X_y + \hat{k}\rho X_z$$

$$\frac{\partial P}{\partial x} = \rho X_x = 0$$

$$\frac{\partial P}{\partial y} = \rho X_y = 0$$

$$\frac{\partial P}{\partial z} = \rho X_z = -\rho g$$

Gravity is the only one body force acting in –ve z direction hence above eq^n.

$\frac{\partial p}{\partial x} = 0 \quad \frac{\partial p}{\partial y} = 0$ Pressure is only function of z only.

for total derivative

$$\boxed{\frac{dp}{dz} = -\rho g}$$

Integration done only when variation of ρ & p & z is known.

For incompressible flow, ρ = constant.

$$p = -\rho g z + c.$$

If $p = p_0 \quad z = z_0$.

$$p_0 = -\rho g z_0 + c.$$

$$c = p_0 + \rho g z_0.$$

$$p - p_0 = \rho g (z_0 - z) = \rho g h$$

$\boxed{p = p_0 + \rho g h}$ Toricelli's Principle.

p_0 = Local atmospheric Pressure.

For isothermal fluid →

If fluid is perfect gas at rest at constant temp. $p = \rho R T \quad \frac{p}{\rho} = \text{constant}$

$$\frac{p}{\rho} = \frac{p_0}{\rho_0}$$

p_0, ρ_0 → Reference states

Let $\frac{dp}{p} = -\rho g$

$$\frac{dp}{p} = -\frac{\rho_0}{p_0} g \, dz$$

$$\ln p = -\frac{\rho_0}{p_0} g z + c.$$

$$c = \ln p_0 + \frac{\rho_0}{p_0} g z_0$$

$$\ln \frac{p}{p_0} = -\frac{\rho_0}{p_0} g (z - z_0)$$

$$\boxed{\frac{p}{p_0} = \exp\left[-\frac{\rho_0}{p_0} g (z - z_0)\right]}$$

Non-isothermal fluid →

The temp. changes upto certain altitude decrease linearly with z.

$$T = T_0 - \alpha z$$

T_0 = absolute temp. at sea level
= 288 K

α = Lapse rate = 6.5 K/km.

$$\frac{dp}{dz} = -\rho g \qquad \rho = \frac{p}{RT} = \frac{p}{R(T_0 - \alpha z)}$$

$$\frac{dp}{dz} = -\frac{p g}{R(T_0 - \alpha z)} \qquad \frac{dp}{p} = -\frac{g}{R} \frac{1}{(T_0 - \alpha z)} dz$$

$$\ln p = \frac{g}{R\alpha} \ln (T_0 - \alpha z) + c$$

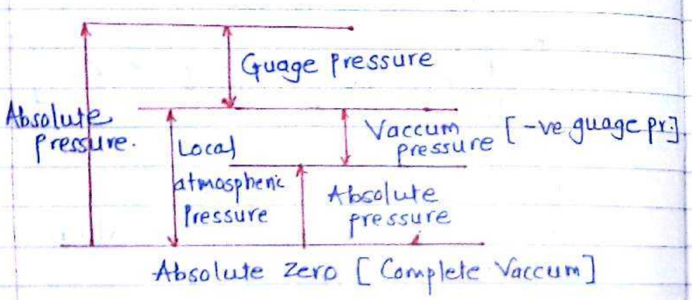
At $z = 0$ $\quad P = P_0$.

$$C = \ln P_0 - \frac{g}{R\alpha} \ln T_0$$

$$\ln \frac{P}{P_0} = \frac{g}{R\alpha} \ln \left(\frac{T_0 - \alpha z}{T_0} \right)$$

$$\boxed{\frac{P}{P_0} = \left(1 - \frac{\alpha z}{T_0} \right)^{g/R\alpha}}$$

Pressure Measurement →



* At sea level, the international standard atmosphere

$$P_{atm} = 101.32 \text{ kN/m}^2$$

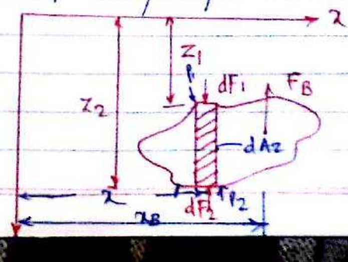
Buoyancy →

Archimedies Principle → This principle states that the buyoant force on a submerged body is equal to the weight of liquid displaced by the body & acts vertically upward through the centroid of the displaced volume.

* Buoyant force depends on the density of the fluid & the submerged volume of the body.

* For a floating body in static equilibrium & in the absence of any other external force, the buoyant force must balance the weight of the body.

When a body is either wholly or partially immersed in a fluid, the hydrostatic lift due to the net vertical component of hydrostatic pressure forces experienced by the body is called the buoyant force & the phenomena is called buoyancy.



Resultant horizontal force is 0
For vertical direction,

$$dF_1 = (p_{atm} + P_1)\, dA_2 = (p_{atm} + \rho g z_1)\, dA_2$$

$$dF_2 = (p_{atm} + P_2)\, dA_2 = (p_{atm} + \rho g z_2)\, dA_2$$

$$dF_B = dF_2 - dF_1 = \rho g (z_2 - z_1)\, dA_2 = \rho g\, d\forall$$

Hence Buoyant force calculated by.

$$F_B = \iiint_{\forall} \rho g\, d\forall = \rho g \forall$$

$$\boxed{F_B = \rho g \forall}$$

$\forall$ = Volume of submerged body
ρ = Density of fluid.

The line of action of the force F_B can be found by taking moment of the force.

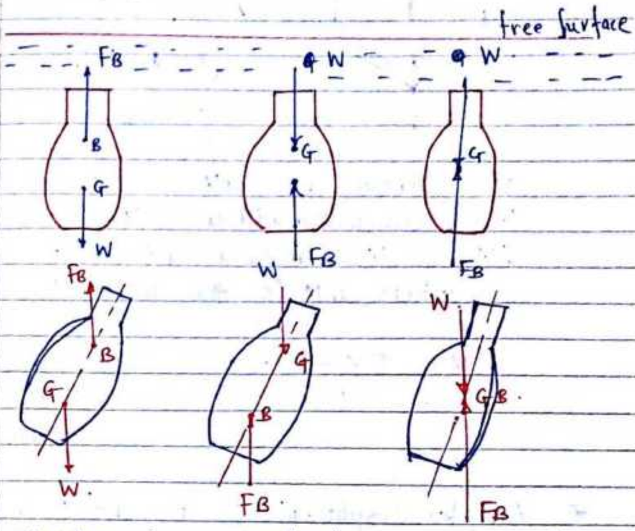
$$x_B F_B = \int x\, dF_B$$

$$x_B \cdot \cancel{\rho g} \forall = \int x \cdot \cancel{\rho g}\, d\forall$$

$$\boxed{x_B = \frac{1}{\forall} \iiint_{\forall} x\, d\forall}$$

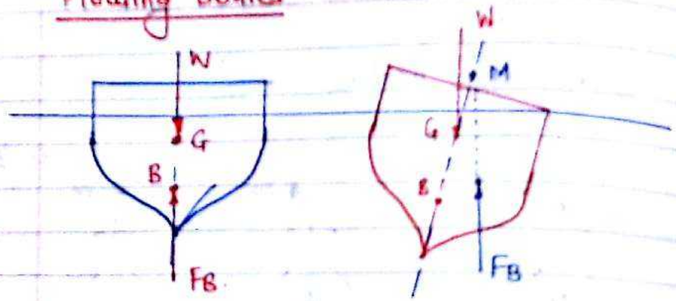
* Stability of unconstrained bodies in fluids →

1) Submerged bodies →



Stable Equilibrium. Unstable Equilibrium Neutral Equilibrium

Floating Bodies →



M is above G $GM > 0$ → stable
M is coinciding with G $GM = 0$ → Neutral
$GM < 0$ M is below G $GM < 0$ → Unstable
Where GM is the metacentric height.

$GM = BM - BG$

where $BM = \frac{I_{min}}{\forall}$ ie. $\frac{I_{yy}}{\forall_{submerged}}$

* Angular displacement of a boat or ship about its longitudinal axis is known as Rolling while that about its transeverse axis is known as pitching.

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* For ships second moment of area about the transerverse axis much greater than that about the longitudinal axis. Hence the stability of a boat with respect to its rolling is much more important than w.r.t ~~to~~ the pitching.

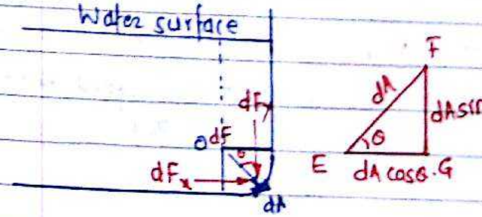
Floating Bodies Containing liquid →

If body carrying liquid like ships with petrol & diesel undergoes ang. displ. the liquid will also move to keep its free surface horizontal. Thus $G^{(c.g.)}$ & as well as B (c.B) changes hence stability of the body is reduced. For this reason liquid which has to be put into no. of compartments so as to minimise its movement within the ship.

* Higher metacentric height higher stability reduces period of roll & hence less comfort.

* Cargo ships → Less GM for comfort.
* Warships & racing yachts → Larger GM. for stability.

* Curved Surfaces →



$dF = P \times Area.$

$dF = \rho g h \times dA$

$F = \int \rho g h \, dA$

Total force $F = \sqrt{F_x^2 + F_y^2}$

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$\tan\phi = \frac{F_y}{F_x}$ inclination with horizontal

$dF_x = dE\sin\theta = \rho g h\, dA \sin\theta$
$dF_y = dF\cos\theta = \rho g h\, dA \cos\theta$

$\therefore$

$F_x = \rho g \int h\, dA \sin\theta \rightarrow$
$F_y = \rho g \int h\, dA \cos\theta$

ΔEFG $EF = dA$ $FG = dA\sin\theta$
$EG = dA\cos\theta$

$F_x \rightarrow$ * $FG = dA\sin\theta = FG$ = vertical projection of the area dA & hence the expression $\rho g \int h\, dA \sin\theta$ represents projected area of the curved surface on vertical plane.

F_y * $h\, dA\cos\theta = EGh \rightarrow$ represent volume of liquid contained above curved surface upto free surface.

$F_x \rightarrow \rho g \bar{h} \times A_p \leftarrow$ Projected area

$F_y \rightarrow \rho V g \rightarrow \rho g \times (A_{c/s} \times W) \rightarrow$ Weight of liquid above curved surface.

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Chapter.

Kinematics of fluid flow

Kinematics → Branch of fluid mechanics which describes the fluid motion & its consequences without consideration of the nature of forces [ie mass of fluid] causing the motion.

Scaler → quantity which has only magnitude. eg. mass, density, Temp.

Vector → quantity which has magnitude & direction to specify. eg. Force, velocity, disp.

Scaler field → If at every point in a region a scaler function has a defined value, the region is called a scaler field. eg. Temp. distribution in a rod.

Vector field → If at every point in a region, a vector function has a defined value the region is called vector field. eg. Force & velocity fields are vector fields.

Methods to describe fluid motion →

1) Langragian method → A single fluid particle is followed during its motion & its velocity Acel^n, density etc. are described.

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2) Eulerian Method → The velocity, accl^n, pressure, density are described at a point in flow field. The Eulerian method is commonly used in fluid mechanics.

<u>Types of flow</u> →

1) Steady flow → Fluid prop. at a point do not change w.r.t. time.

$$\left(\frac{\partial v}{\partial t}\right)_s = 0 \quad \left(\frac{\partial p}{\partial t}\right)_s = 0 \quad \left(\frac{\partial \rho}{\partial t}\right)_s = 0$$

2) Unsteady flow → Fluid prop. at a point changes w.r.t time.

3) Uniform Flow → Fluid prop. like velocity at any given time doesn't change w.r.t space

$$\left(\frac{\partial v}{\partial s}\right)_t = 0$$

4) Nonuniform flow → $\left(\frac{\partial v}{\partial s}\right)_t \neq 0$

5) Compressible flow → Density of the fluid changes from point to point

$$\rho \neq \text{constant}.$$

6) Incompressible flow → $\rho = \text{constant}$.

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7) Laminar flow → flow in which fluid particles move along well defined path or stream lines are straight & parallel.

8) Turbulent flow → fluid particles moves in zig-zag way, formation of eddies & energy loss takes place.

9) Rotational flow → fluid particle flowing through streamline also rotates about their own axis.

10) Irrotational flow → Do not rotate while flowing through streamline.

* Material Derivative and accleration →
Position of particle at any instant t in a flow field given by space co-ordinates (x, y, z) w.r.t. rect. cartesian frame of reference. The velocity component u, v, w along x, y, z directions resp.

In the Eulerian form

$$u = u(x, y, z, t)$$
$$v = v(x, y, z) t)$$
$$w = w(x, y, z, t)$$

For small time interval Δt particle moves by $x + \Delta x$, $y + \Delta y$, $z + \Delta z$

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By Taylors series we can write,

$$\frac{\Delta u}{\Delta t} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} + H.O.T.$$

$$\frac{\Delta v}{\Delta t} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} + H.O.T.$$

$$\frac{\Delta v}{\Delta t} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} + H.O.T.$$

If $\Delta t \to 0$ eq^{n}s becomes.

$$\left[\lim_{\delta t \to 0} \frac{\Delta u}{\Delta t} = \frac{Du}{Dt}\right]$$

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$\frac{Dw}{Dz} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

* Total differential $\frac{D}{Dt}$ is known as material or substantial derivative w.r.t. time.

* The term $\frac{\partial u}{\partial t} \Rightarrow$ temporal or local acc^n.

* $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \Rightarrow$ Convective accleration.

Type of flow	Material Accleration Temporal / Local	Convective
1) steady & uniform	0	0
2) steady & non uniform	0	Exists.
3) Unsteady & uniform	Exists	0
4) Unsteady & Non-uniform	Exists	Exists.

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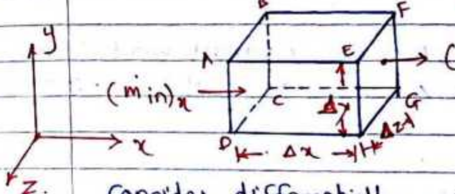
Control mass $\to$ closed system $\to$ $\dot{m} = 0$

Conservation of mass for fluid flow $\to$

A fluid being a material body, must obey the law of conservation of mass in the course of its flow. If a velocity field $\vec{v}$ has to exist in fluid continum, the velocity components must obey mass cons. principle.

Continuity equation from Eulerian point of view or Control volume approach $\to$ open system.



Consider differentially small rect. control volume. Let u velocity $\dot{m}$ mass rate entering in x dir. ie. face ABCD

$$(\dot{m}_{in})_x = \rho u \Delta y \times \Delta z$$

Mass rate leaving from $x + \Delta x$ ie. face EFGH.

$$(\dot{m}_{out})_{x+\Delta x} = (\dot{m}_{in})_x + \frac{\partial}{\partial x}(\dot{m}_{in})_x \cdot \Delta x + H.O.T.$$

$$= \rho u \Delta y \Delta z + \frac{\partial}{\partial x} \rho u \Delta y \Delta z \cdot \Delta x + H.O.T$$

Net rate of mass entering the control volume

$$(\dot{m}_{in})_x - (\dot{m}_{out})_{x+\Delta x} = \frac{-\partial}{\partial x} \rho u \Delta x \Delta y \Delta z + H.O.T. \quad -(1)$$

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In similar fashion we can write for y & z direction also.

$$(\dot{m}_{in})_y - (\dot{m}_{out})_{y+\Delta y} = -\frac{\partial}{\partial y} \rho v \, \Delta x \Delta y \Delta z + H.O.T \quad -(2)$$

$$(\dot{m}_{in})_z - (\dot{m}_{out})_{z+\Delta z} = -\frac{\partial}{\partial y} \rho z \, \Delta x \Delta y \Delta z + H.O.T. \quad -(3)$$

Conservation of mass for control volume

$$\dot{m}_{in} - \dot{m}_{out} = \frac{\partial}{\partial t}(m_{cv}) = \frac{\partial}{\partial t}(\rho \Delta x \Delta y \Delta z)$$

$\rho = \frac{m}{V}$
$m = \rho V$

$$= \frac{\partial \rho}{\partial t} \Delta x \Delta y \Delta z. \quad -(4)$$

Since $\Delta x, \Delta y, \Delta z$ invariant w.r.t. time

Taking $\Delta x, \Delta y, \Delta z \to 0$ $\quad H.O.T \to 0$

Equating 1 2 3 & 4

$$-\frac{\partial}{\partial x} \rho u \Delta x \cancel{\Delta y} \Delta z - \frac{\partial}{\partial y} \rho v \Delta x \cancel{\Delta y} \Delta z - \frac{\partial}{\partial z} \rho w \Delta x \cancel{\Delta y} \Delta z$$

$$= \frac{\partial \rho}{\partial t} \Delta x \cancel{\Delta y} \Delta z$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \rho u + \frac{\partial}{\partial y} \rho v + \frac{\partial}{\partial z} \rho w = 0$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0} \quad -(5)$$

.... General form of continuity eq^n

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Where $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$.

$\vec{V} = \hat{i} u + \hat{j} v + \hat{k} w$.

1) For incompressible flow, ρ = constant w.r.t. space as well as time.
Hence eq^n 5 becomes.

$$\boxed{\nabla \cdot \vec{V} = 0} \quad \boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

2) For steady flow, eq^n (5) becomes

$$\boxed{\nabla \cdot (\rho \vec{V}) = 0}$$

$$\boxed{\frac{\partial}{\partial x} \rho u + \frac{\partial}{\partial y} \rho v + \frac{\partial}{\partial t} \rho w = 0}$$

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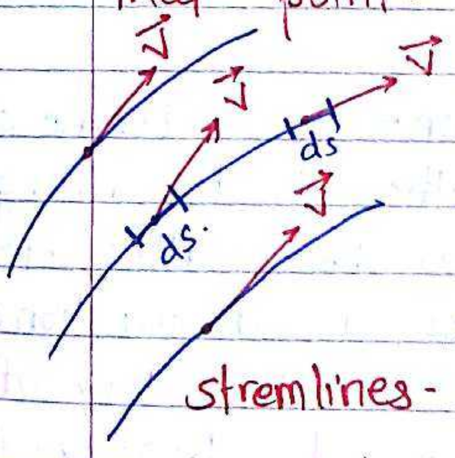
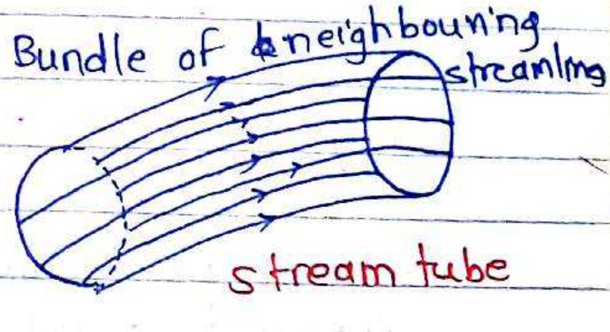
streamlines →

The analytical description of flow velocities by the Eulerian approach is geometrically depicted through the concept of streamlines. If for a fixed instant of time, a space curve is drawn so that it is tangent everywhere to the velocity vector, then this curve is called streamline. Therefore the Eulerian method gives a series of instantaneous streamlines of the state of motion. In other words, a streamline at any instant can be defined as an imaginary curve or line in a flow field so that the tangent to the curve at any point represents the direction of the instantaneous velocity at that point.



stremlines -



$d\vec{s} \times \vec{V} = 0$ [Angle between two vectors 0 ie. Cross product 0]

$$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$d\vec{s} \times \vec{V} = 0 \quad \text{ie.}$$

Note → The two streamlines can never intersect each other since the instantaneous velocity vector at a given location is always unique.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & dy & dz \\ u & v & w \end{vmatrix} = 0$$

$$\hat{i}(w\,dy - v\,dz) - \hat{j}(w\,dx - u\,dz) + \hat{k}(v\,dx - u\,dy) = 0$$

$w\,dy - v\,dz = 0 \qquad w\,dx - u\,dz = 0$

$v\,dx - u\,dy = 0$

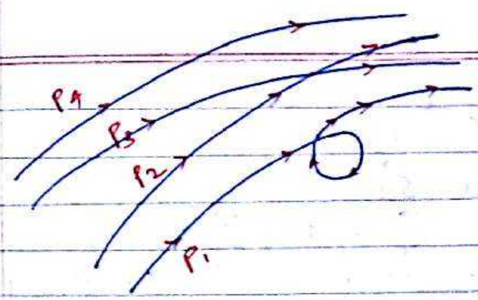
$w\,dy = v\,dz \qquad w\,dx = u\,dz$

$v\,dx = u\,dy$

$$\frac{dy}{v} = \frac{dz}{w} \qquad \frac{dx}{u} = \frac{dz}{w} \qquad \frac{dx}{u} = \frac{dy}{v}$$

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}} \text{ — Eq}^n \text{ of streamline.}$$

Pathlines → Pathline is outcome of the Langrangian method in describing fluid flow and show the paths of different fluid particles as a function of time. A pathline is a trajectory of a fluid particle of fixed identity. Therefore a family of pathlines represents the trajectories of different particles say P_1, P_2, P_3.



The description of pathlines inherently involves the variation of time, since a fluid particle takes time to move from one point to another. Two pathlines can intersect with one another or a single pathline itself can form a loop.

Note For steady flow, pathlines are identical to streamline in as steady the Eulerian and the Langrangian versions becomes the same.

Streakline → A streakline at any instant of time is the locus of the temporary locations of all particles that have passed through a fixed point in the flow field. If a dye injected into a liquid at a fixed point in the flow field, then at a later time t the dye will indicate the end points of the path lines of particles which have passed through the injection point.

By Langangian method, if a fluid particle $(\vec{S_0})$ at time t can be derived which passes through a fixed point $(\vec{S_i})$

then $S(\vec{S_0}, t) = \vec{S_1}$

solving for $\vec{S_0}$

$$\vec{S_0} = F(\vec{S_1}, t) \quad -①$$

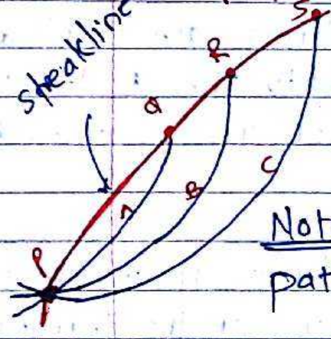
If the positions $(\vec{S})$ of the particles which have passed through the fixed point $(\vec{S_1})$ are determined, then a streakline can be drawn through these point.

eq^n of streakline

$$\vec{S} = f(\vec{S_0}, t)$$

from ①

$$\vec{S} = f[F(S_1 t)\, t]$$



Note For steady flow streamline, pathlines & streaklines are identical.

stream function $(\psi) \rightarrow$

The flow field is such that

$$u = u(x, y, t) \quad v = v(x, y, t)$$
$$w = 0$$

The eq^n of continuity is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

If a function $\psi(x, y, t)$ is defined in a manner $u = \frac{\partial \psi}{\partial y}$ $\quad v = -\frac{\partial \psi}{\partial x}$

so that it automatically satisfies the eq^n of continuity. Then the function ψ is known as stream function.

For 2D irrotational flow.

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0.$$

$$\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) = 0$$

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

Note For irrotational flow stream function satisfies the Laplace equation.

Change in stream function $d\psi$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$
$$= -v\,dx + u\,dy$$
$$d\psi = u\,dy - v\,dx.$$

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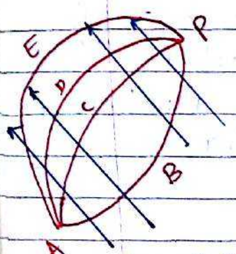
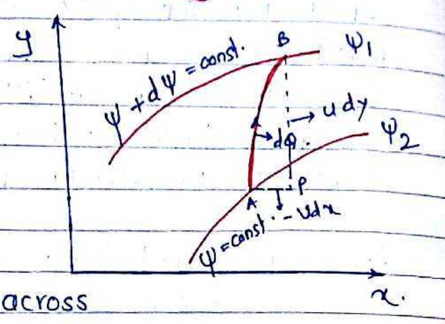
Eq^n of streamline

$$\frac{u}{dx} = \frac{v}{dy}$$

$$u\,dy - v\,dx = 0.$$

ie. on streamline $d\psi = 0$ ie. the value of ~~str.~~ ψ along streamline is constant.





The rate of flow across any curve between A & P depends only on the end points A & P.

* The volume of fluid crossing the surface AB must be flowing out from surfaces AP & BP of unit width. Hence.

$$dQ = dy \times 1 \times u - v \times dx \times 1$$
$$= u\,dy - v\,dx$$
$$dQ = d\psi = \psi_1 - \psi_2$$

* Difference between two ~~str~~ stream function gives volume flow rate.

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Velocity Potential → Irrotationality leads to the condition $\nabla \times \vec{V} = 0$ which demands $\vec{V} = \nabla\phi$ where ϕ is known as a potential function.

$$\vec{V} = \nabla\phi_{\text{scaler}}$$
$$= \text{gradient of scaler function}$$

$$u = \frac{\partial\phi}{\partial x} \quad v = \frac{\partial\phi}{\partial y} \quad w = \frac{\partial\phi}{\partial z}.$$

2D incompressible flow continuity eq^n

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Substitute u, v, w

$$\boxed{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0}$$

... Laplace eq^n

Relationship between velocity potential & stream function → (ψ, ϕ)

Consider 2D, incomp. & irrot. flow so that both function ψ & ϕ exist.

$$\phi = \phi(x, y) \quad \text{— 2D flow.}$$

$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy$$

$$d\phi = u\,dx + v\,dy$$

For constant ϕ $\quad d\phi = 0.$

$$\left.\frac{dy}{dx}\right|_{\phi\ \text{const.}} = -\frac{u}{v}$$

Scanned by CamScanner

2D incomp. stream function.

$$\psi = \psi(x,y)$$

$$d\psi = \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy$$

$$0 = -v\,dx + u\,dy \quad \ldots\, d\psi = 0 \text{ for } \psi = \text{const}$$

$$\left.\frac{dy}{dx}\right|_{\psi\,\text{const}} = \frac{v}{u}$$

$$\left.\frac{dy}{dx}\right|_{\phi\,\text{const}} \times \left.\frac{dy}{dx}\right|_{\psi\,\text{const}} = \frac{-u}{v} \times \frac{v}{u} = -1$$

(where $u, v \neq 0$)

Note Implies that the lines of constant ϕ (equipotential lines) & lines of constant ψ are orthogonal to eachother everywhere in the flow field except at certain points where the velocities are zero i.e. stagnation points.

* A fluid motion consists of translation, rotation and continuous deformation. In a uniform flow, the fluid elements are simply translated without any deformation or rotation. The deformation & rotation of fluid element are caused by the variations in velocity components with the space co-ordinate.

Rate of linear deformation / strain rate → Rate of change of length of linear fluid element per unit original length.

$$\dot{\varepsilon}_x = \frac{\partial u}{\partial x} \quad \dot{\varepsilon}_y = \frac{\partial v}{\partial y} \quad \dot{\varepsilon}_z = \frac{\partial w}{\partial z}$$

Volumetric strain → Rate of change of volume per unit original volume

$$\dot{\varepsilon}_{vol} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$$= \dot{\varepsilon}_x + \dot{\varepsilon}_y + \dot{\varepsilon}_z$$

Volume of fluid element is denoted by $\forall$ $\quad \dot{\varepsilon}_{vol} = \frac{1}{\forall}\frac{D\forall}{Dt}$ ... By def.

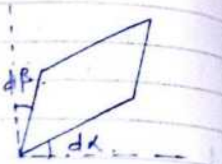
$$\frac{D\forall/Dt}{\forall} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla\cdot\vec{V}$$

$\dot{\varepsilon}_{vol}$ = Divergence of velocity vector.

No volumetric strain for incomp. flow hence $\boxed{\nabla\cdot\vec{V} = 0}$ ... In comp. flow

* Rate of angular deformation $\rightarrow (\dot{\varepsilon}_{xy})$ defined as rate of change of angle between two line elements in the fluid which were originally perpendicular to each other.

$$\dot{\varepsilon}_{xy} = \frac{d\alpha}{dt} + \frac{d\beta}{dt} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$



$$\dot{\varepsilon}_{yz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

$$\dot{\varepsilon}_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

* Rotation of fluid element $\rightarrow$ The rotation of fluid element in the absence of any deformation is known as pure or rigid body rotation.

Rotation is defined as Arithmatic mean of the angular velocities of two perpendicular linear meeting at that point.

$$\omega_{xy} = \frac{1}{2}(\dot{\alpha} - \dot{\beta}) = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \omega_z.$$

$$\omega_{yz} = \omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_{zx} = \omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

Rotation in a flow field can be expressed in vector form.

$$\boxed{\vec{\omega} = \frac{1}{2}(\nabla \times \vec{V})}$$

$\Omega = \nabla \times \vec{V} \Rightarrow$ Vorticity of flow.

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}.$$

$$\Omega = \nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$= \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$$

$$\boxed{\Omega = 2\vec{\omega}}$$

If an imaginary line is drawn in the fluid element so that the tangent to it at each point is in the direction of the vorticity vector $\vec{\Omega}$ at that point the line is called a vortex line.

* For Irrotational flow $\vec{\omega} = 0$ $\quad \frac{1}{2}(\nabla \times \vec{V} = 0)$

$$\boxed{\nabla \times \vec{V} = 0}$$ Irrotational flow

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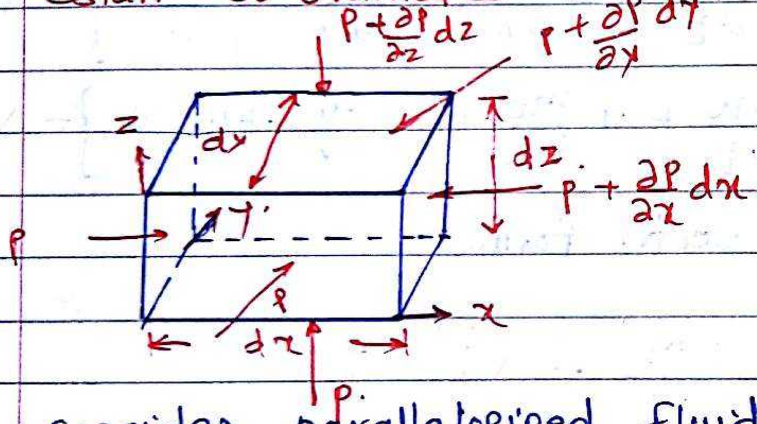
Dynamics of inviscid flow

Dynamics → In this, role of some of the forcing parameters that influence the motion as well as their relation with the motion of fluid.

* Equation of motion for inviscid flow in cartesian co-ordinates →



Consider parallelopiped fluid element as control mass system [closed system]

Let b_x, b_y, b_z → Body forces acting per unit mass of fluid along x, y, z dir.

Newtons second law of motion in x dir.

$$\Sigma F_x = (dm)\, a_x$$

As fluid is ideal hence $\mu = 0$ ie. no shear stress only normal stress ie. -ve fluid Pressure.

$$\underbrace{p\,dy\,dz - \left(p + \frac{\partial p}{\partial x}dx\right)}_{\text{Surface force}} + \underbrace{\rho\, b_x\, dx\,dy\,dz}_{\text{Body force}} = \underbrace{\rho\, dx\,dy\,dz}_{\text{Mass}}\ \underbrace{\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right]}_{\text{Acceleration}}$$

After simplification

$$-\frac{\partial p}{\partial x} + \rho b_x = \rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right]$$

$$\boxed{\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = \rho b_x - \frac{\partial p}{\partial x}} \quad \text{—(1)}$$

→ Eulers equation of motion

y, z direction.

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right] = \rho b_y - \frac{\partial p}{\partial y} \quad \text{—(2)}$$

$$\rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right] = \rho b_z - \frac{\partial p}{\partial z} \quad \text{—(3)}$$

Single vector form.

$$\rho\frac{D\vec{V}}{Dt} = \rho\vec{b} - \nabla p$$

or $$\boxed{\rho\left[\frac{\partial V}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}\right] = \rho\vec{b} - \nabla p}$$

* Pressure differential between 2 points in an inviscid flow field.

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz$$

After putting values of $\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}$ from (1) & (2) (3) & vector simplication we get. & considering body force only gravity acting along −ve z direction ie. $b_x = 0$ $b_y = 0$ $b_z = -g$.

$$\boxed{dp + \frac{1}{2}\rho\, dV^2 + \rho g\, dz + \rho\left[\frac{\partial\vec{V}}{\partial t}d\vec{l}\right] - \rho \times [(\vec{V}\times\vec{\xi})\cdot d\vec{l}] = 0}$$

$d\vec{l} \Rightarrow$ Position vector $= dx\hat{i} + dy\hat{j} + dz\hat{k}$

$\vec{\xi} \Rightarrow$ Vorticity $\vec{\xi} = \nabla\times\vec{V}$

Case 1 → when $d\vec{l}$ is along streamline $\vec{V}$ & $d\vec{l}$ are oriented in same direction $(\vec{V}\times\vec{\xi})\,d\vec{l} = 0$

Case 2 → when $\vec{\xi} = 0$ (irrotational flow)

Case 3 → $\vec{V}\times\vec{\xi}$ is perpedicular to $d\vec{l}$

the above eqn becomes.

$$dp + \frac{1}{2}\rho\, dV^2 + \rho g\, dz = 0$$

divide by ρ.

$$\boxed{\frac{dp}{\rho} + \frac{1}{2}dV^2 + g\,dz = 0}$$

Integrating above eqn. from 1 - 2

$$\int_1^2 \frac{dp}{\rho} + \int_1^2 \frac{1}{2}dV^2 + \int_1^2 g\,dz = 0$$

If ρ = constant

$$\frac{p_2 - p_1}{\rho} + \frac{1}{2}[V_2^2 - V_1^2] + g(z_2 - z_1) = 0$$

$$\boxed{\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2}$$

.... Bernoulli's equation.

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Assumptions →

1) Flow is inviscid ie. $\mu = 0$
2) steady flow.
3) Incompressible flow [Fluid may be compressible or incompr.]
4) Points 1 & 2 are located on streamline
5) Flow field is irrotational.
6) $\vec{V} \times \vec{\xi}$ is perpendicular to $d\vec{l}$
7) No work transfer 8) No heat transfer.

eq^n can be written as:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2.$$

1) $z = \frac{mgz}{mg} = \frac{\text{Potential energy}}{\text{weight}}$ = Potential head

2) $\frac{\frac{1}{2}V^2}{g} = \frac{\frac{1}{2}mV^2}{mg} = \frac{\text{Kinetic energy}}{\text{weight}}$ = kinetic head

3) Flow energy/ work $= F \times \Delta x = PA\Delta x$

$\frac{\text{Flow energy}}{\text{weight}} = \frac{PA\Delta x}{\rho \forall g} = \frac{P\forall}{\rho \forall g} = \frac{P}{\rho g}$ = Pressure head

$$\boxed{\frac{P}{\rho g} + \frac{V_1^2}{2g} + Z = \text{constant}}$$

... Conservation of mechanical energy.

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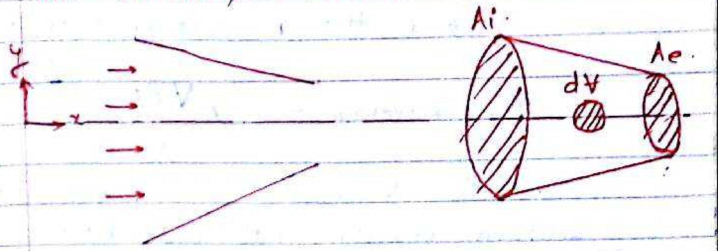
One dimensional c/s nally Averaged form of the continuity equation.



Consider steady flow through a variable area conduit.

$\nabla \cdot \rho\vec{V} = 0$ ... for steady flow.

Consider elemental volume $d\forall$ integrating over domain

$$\int_{\forall} \nabla \cdot \rho\vec{V}\, d\forall = 0$$

Using divergence theorem volume integral conv into area that bounds volume.

$$\int_A (\rho\vec{V})\hat{n}\, dA = 0$$

$\hat{n} \rightarrow$ Unit vector normal to dA

Since velocity vector is zero except inflow & outflow.

$$\int_{Ai} (\rho\vec{V})\hat{n}\, dA + \int_{Ae} \rho\vec{V}\hat{n}\, dA = 0$$

Since one unit normal vector towards velocity vector & one opposite

$\hat{n} = -\hat{i}$ over Ai $\quad \hat{n} = +\hat{i}$ over Ae

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$$-\int_{Ai} \rho V dA + \int_{Ae} \rho V dA = 0 \quad -①$$

$V \Rightarrow$ is the x component of flow velocity.

$$V_{average} = \frac{\int_A V dA}{A}$$

Physically $V_{average}$ is an equivalent uniform velocity that could have given rise to the same volumetric flow rate as that induced by the variable velocity under consideration

$$\bar{V} = \frac{\int_A V dA}{A} \quad -②$$

Combining eq^n ① & ②

$$\rho_i \bar{V}_i A_i = \rho_e \bar{V}_e A_e$$

If ρ = constant [incomp. flow]

$$\boxed{A_i \bar{V}_i = A_e \bar{V}_e}$$

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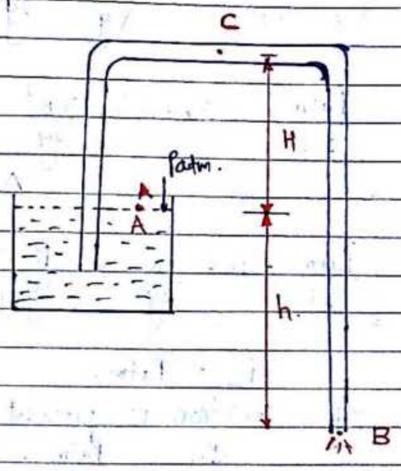
Hydraulic Siphon $\rightarrow$ Applications of Bernaulli's eq^n.



Fluid always flow from a higher total mechanical energy level to lower energy level.
Applying Bernaulli's eq^n at pt A & B

$$\frac{\cancel{P_A}}{\rho g} + \frac{\cancel{V_A^2}}{2g} + Z_A = \frac{\cancel{P_B}}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_f.$$

Patm, negligible, Patm.

$$Z_A - Z_B = \frac{V_B^2}{2g} + h_f.$$

$$\sqrt{2g(h-h_f)} = V_B$$

$$\boxed{V_B = \sqrt{2gh}} \ \ldots \text{ if } h_f = 0.$$

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at point A & C.

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_c}{\rho g} + \frac{V_c^2}{2g} + Z_c$$

$$\frac{P_{atm}}{\rho g} + (Z_A - Z_C) - \frac{V_B^2}{2g} = \frac{P_c}{\rho g}$$

$\because$ As $= \underline{\underline{V_c = V_B}}$

$$\boxed{\frac{P_{atm}}{\rho g} - H - \frac{V_B^2}{2g} = \frac{P_c}{\rho g}}$$

$P_c < P_{atm}$.

If friction is present

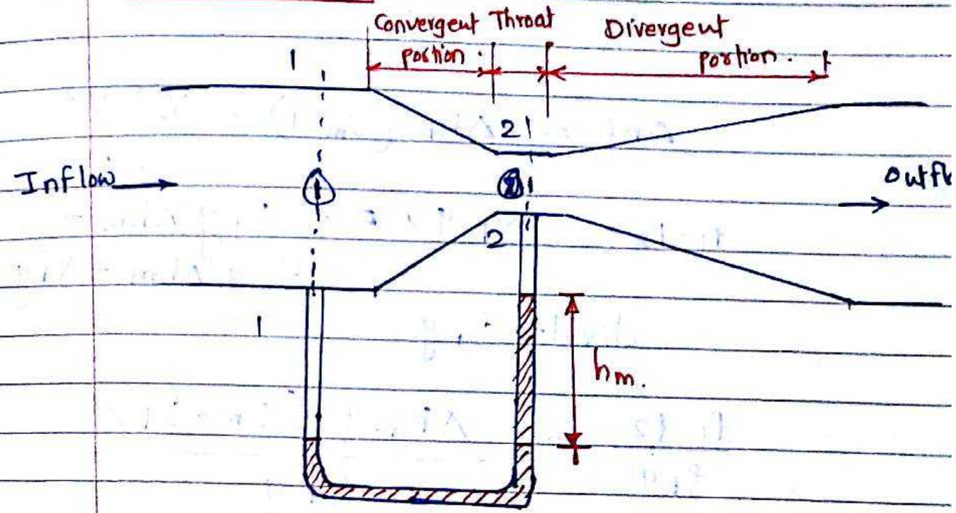
$$\frac{P_c}{\rho g} = \frac{P_{atm}}{\rho g} - H - h_f - \frac{V_B^2}{2g}$$

* But for a siphon $P_c > P_{min}$ where is the pressure for air locking or vapour locking to start as If the pressure of a liquid becomes equal to its vapour pressure at the existing temp then the liquid starts boiling & creates problem of cavitation.

* For water this min pressure is about 20 kPa (2 m of water).

Measurement of flow rate → (through pipes)

i) Venturimeter →



Convergent passage is short & divergent is long ensures rapid flow in conv. portion & gradual flow to avoid loss of energy due to sepration of flow at walls in divergent portion.

Converging Cone angle ⇒ 15-20°

Divergent cone angle ⇒ 5-7°

Velocity reaches ~~at~~ max. at the throat according principle of continuity & pressure decreases at it min value acco. to Bernaulli's equation.

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at point A & C.

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_c}{\rho g} + \frac{V_c^2}{2g} + Z_c$$

$$\frac{P_{atm}}{\rho g} + (Z_A - Z_C) - \frac{V_B^2}{2g} = \frac{P_c}{\rho g}$$

$\therefore$ As $= \underline{\underline{V_c = V_B}}$

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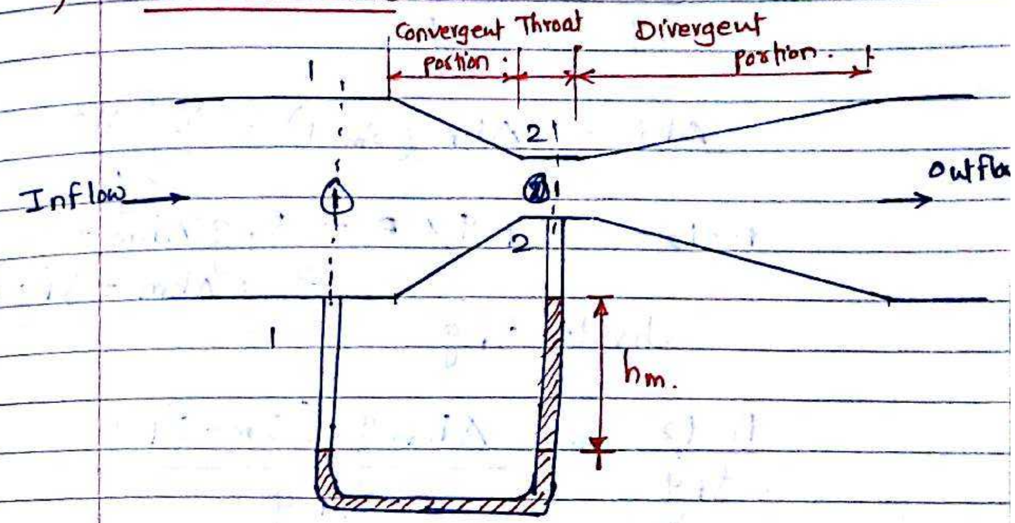

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Bernaulli's eq^n at pt 1 & 2

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$z_1 = z_2$

$$\frac{\Delta p}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$\Delta h_f = \Delta h_m (S_m - 1) = \frac{V_2^2 - V_1^2}{2g}$$

$$p_1 - p_2 = -\rho_f g x - \rho_f g \Delta h_m + \rho_m g \Delta h_m + \rho_f g x$$

divide $\rho_f g$.

$$\frac{p_1 - p_2}{\rho_f g} = \frac{\Delta h_m g (\rho_m - \rho_f)}{\rho_f g}$$

$$\boxed{\frac{\Delta p}{\rho g} = \Delta h_m \left(\frac{\rho_{manometric}}{\rho_{fluid}} - 1 \right)}$$

$\rho_{fluid} = \rho_{water}$

$$\boxed{\frac{\Delta p}{\rho g} = \Delta h_m (S_m - 1)}$$

$$\boxed{Q = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}}$$

where $h = \left[\frac{\rho_m}{\rho_{fluid}} - 1 \right] \Delta h$.

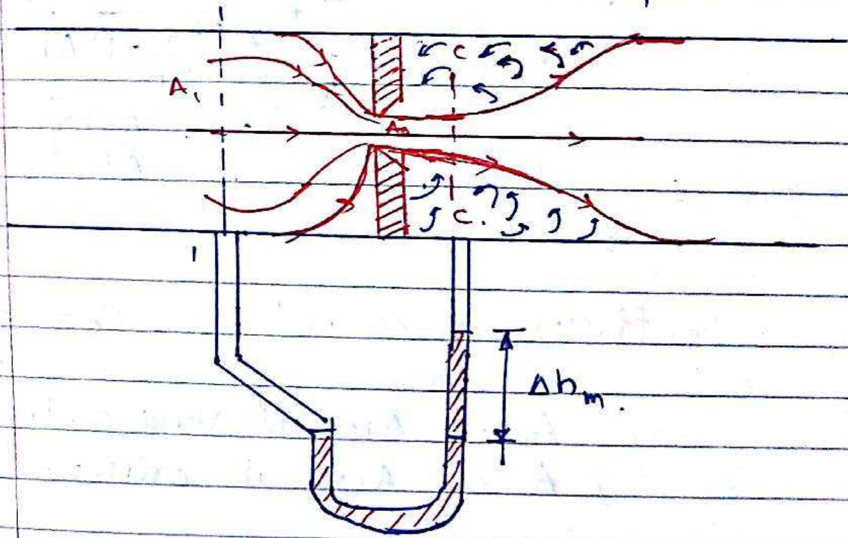
Because of frictional losses in addition to the change in momentum the eq^n of Q always overestimates the actual flow rate

Coefficient of discharge $C_d = \frac{\text{Actual Discharge}}{\text{Theor. Discharge}}$

* C_d for Venturimeter lies between 0.95 to 0.98.

2) Orificemeter → Simpler & cheaper arrangement of flow. Orifice is thin circular plate sharpe edged concentric plate circular hole on it $A_0 <<< A_1$

$A_0 \Rightarrow$ Area of orifice



Flow from upstream contracts till vena contracta achieved.

Section 1-1 & C,C

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_c}{\rho g} + \frac{V_c^2}{2g}.$$

$$A_1V_1 = A_cV_c$$

$$V_c = \sqrt{\frac{2(P_1 - P_c)}{\rho\left(1 - \frac{A_c^2}{A_1^2}\right)}}$$

Cv → Coeff. of velocity is determined to get Vactual as friction losses are present

$$V_c = C_v \cdot \sqrt{\frac{2g\left(\frac{\rho_m}{\rho_f} - 1\right)\Delta h}{\left(1 - \frac{A_c^2}{A_1^2}\right)}}$$

$$Q = A_c V_c$$

Coefficient of contraction $= C_c = \dfrac{A_c}{A_o}$

A_c = Area at vena contracta.
A_o = Area at orifice.

Cd for orifice 0.60 to 0.65

$$Q = \underbrace{C_c A_o C_v \sqrt{\frac{2g}{\left(1 - \frac{C_c^2 A_o^2}{A_1^2}\right)}}}_{C} \sqrt{\left(\frac{\rho_m}{\rho} - 1\right)\Delta h}$$

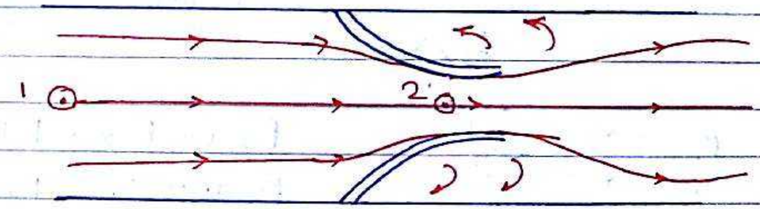
$$\boxed{Q = C_d A_o \sqrt{\frac{2gh}{\left(1 - \frac{C_c^2 A_o^2}{A_1^2}\right)}}}$$

where $C_d = C_v \cdot C_c$.

$h = \left(\frac{\rho_m}{\rho} - 1\right)\Delta h$

C depends upon orifice/duct area, Reynolds number.

3) Flow nozzle → Cd → 0.70 to 0.80



It's venturimeter with the divergent part omitted. Therefore basic calcⁿ of flow rate are the same as those for a venturimeter. The dissipation of energy downstream of the throat due to flow seperation is greater than that for a venturimeter. Lowest cost of nozzle than venturimeter.

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Static Pressure → The thermodynamic or hydrostatic pressure caused by molecular collisions is known as static pressure in a fluid flow.

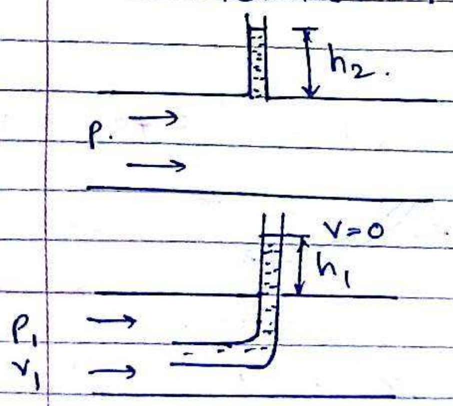
$$\frac{P}{\rho g} + \frac{V^2}{2g} + gz = C$$

multiply by ρg.

$$P + \frac{V^2}{2g} \times \rho g = C$$

$$P + \frac{1}{2}\rho V^2 = P_0 \text{ Total pressure}$$

$P_{static} + P_{\cancel{stagnation}}$ dynamic $= P_{Total}$ or stagnation pressure.



$P_{static} = \rho g h$.

$P_{sta} + P_{dy} = P_{stag}$.

$P_{stat} + 0 = P_{stagnation}$

$P_{static} - P_{stagnation}$

Stagnation pressure → The stagnation pressure at a point in a fluid flow is the pressure which could result if the fluid were brought to rest isentropically. Isentropic means entire kinetic energy of fluid particle is utilised to increase its pressure energy. This is only possible in rev. adiabatic or isentropic process only.

Let $P + \frac{1}{2}\rho V^2 = P_0$

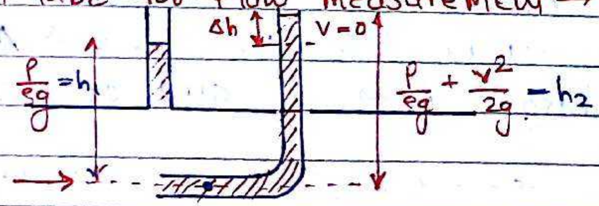
$$V = \sqrt{2(P_0 - P)/\rho}$$

$$V = C\sqrt{2\left(\frac{P_0 - P}{\rho}\right)}$$

$C \Rightarrow$ empirical factor account for part of K.E will be converted into inter molecular energy, due to fluid friction.

$$\boxed{V = C\sqrt{2g\left(\frac{\Delta P}{\rho g}\right)}}$$

Pitot tube for flow measurement →



$P_{static} = \rho g h_1$

$P_{\substack{\cancel{dynamic} \\ stag.}} = \rho g h_2$ $\quad \underline{\underline{V=0}}$

$$\rho g h_1 + \frac{1}{2}\rho V^2 = \rho g h_2$$

$$\frac{1}{2}V^2 = gh_2 - gh_1$$

$$V = \sqrt{2g\Delta h}$$

* Viscous Flows through Pipes

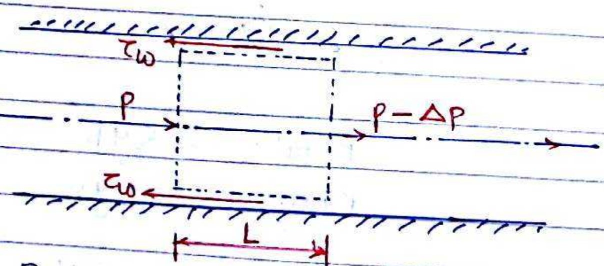
One of the most important items of information that a hydraulic engineer needs is the power required to force fluids at a certain steady rate through some routine solution of pipe flow ~~probl.~~ or pipe network system.

Friction factor in a pipe flow →

Skin friction coefficient for a fully developed flow through a closed duct is defined as

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho V^2} \Rightarrow \tau_w = \frac{1}{2} C_f \rho V^2 \quad —(1)$$

Where, V is the average velocity of flow given by $Q = AV$, $V = Q/A$, Q is discharge. $A \Rightarrow$ c/s area of duct.



Force balance for fluid element

$$\text{shear stress} = \frac{\text{Shear force}}{\text{Shear Area}}$$

$$\tau_{wall} = \frac{\Delta P \times A}{S\,L} \quad —(2)$$

Where $\tau_w \Rightarrow$ Shear stress at the wall
$\Delta P \Rightarrow$ Piezometric pressure drop over length L
$A \Rightarrow$ c/s area
$S \Rightarrow$ Wetted perimeter.

From eq^n (1) & (2)

$$\frac{1}{2} C_f \rho V^2 = \frac{\Delta P \times A}{S\,L}$$

$$C_f = \frac{\Delta P \times A}{S L \frac{1}{2}\rho V^2}$$

$A_{c/s} = \frac{\pi}{4} D_h^2 \qquad S = \pi D_h$

$$C_f = \frac{1}{4} \frac{D_h}{L} \frac{\Delta P}{\frac{1}{2}\rho V^2}$$

$$\boxed{D_h = \frac{4A}{S} = \frac{4 \times \text{c/s area}}{\text{Wetted perimeter}} = \text{Hydraulic Dia.}}$$

To do away with the factor $\frac{1}{4}$. Darcy defined friction factor f

$$f = \frac{D_h}{L} \frac{\Delta P}{(\frac{1}{2})\rho V^2}$$

Where $\boxed{f = 4 C_f}$

* Darcy's friction factor (f) is 4 times of fanning

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$$\Delta P = f \frac{L}{D} \frac{1}{2} \rho V^2$$

Head loss (energy loss per unit weight)

$$h_f = \frac{\Delta P}{\rho g} = f \frac{L}{D} \left(\frac{V^2}{2g}\right)$$

$$\boxed{h_f = \frac{f L V^2}{2 g D}}$$ ...... Darcy weisbach formula.

h_f = Head loss due to friction over the length L of the pipe.

Hydraulic Diameter →

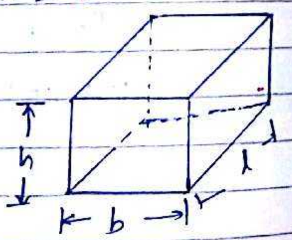
1) For circular c/s

$$D_h = \frac{4 A_{c/s}}{P_{wetted}}$$

$$= \frac{4 \times \pi D^2}{4 \times \pi D}$$

$$\boxed{D_h = D}$$

2) Rectangular c/s



$$D_h = \frac{4 A_{c/s}}{P}$$

$$= \frac{4 \times b \times h}{2 \times (b + h)} = \frac{2bh}{b+h}$$

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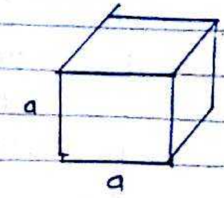
3) Square c/s.



$$= \frac{4 \times a^2}{4a}$$

$$\boxed{D_h = a}$$

Note For circular pipe

Reynold's Number	Flow
$Re \leq 2300$ →	Laminar
$2300 < Re < 4000$ →	Transition
$4000 > Re$ →	Turbulent

where $Re = \frac{\rho V_{avg} . D_h}{\mu}$

Note Friction f depends Reynolds number

$$f = \frac{16}{Re} \quad \text{For } Re < 2000$$ (viscous flow)

$$f = \frac{0.079}{Re^{1/4}} \quad \text{for } Re \text{ varying } 4000 \text{ to } 10^6$$

* Chezy's formula for loss of head due to friction in pipes →

Let, $h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2$

where f' = coefficient of friction

A = c/s area
P = wetted perimeter
V = mean velocity of flow
L = Length of pipe.

* The ratio $\frac{A}{P}$ ⇒ Hydraulic mean depth or hydraulic radius denoted by m.

For circular pipe $= \frac{A}{P} = \frac{\pi/4 d^2}{\pi d} = \frac{d}{4} = m.$

$\therefore \frac{P}{A} = \frac{1}{m}.$

$$hf = \frac{f'}{\rho g} \times L \times V^2 \times \frac{1}{m}$$

$$V^2 = hf \times \frac{\rho g}{f'} \times m \times \frac{1}{L}$$

$$= \frac{\rho g}{f'} \times m \times \frac{hf}{L}$$

$$V = \sqrt{\frac{\rho g}{f'}} \sqrt{m \frac{hf}{L}}$$

$$\boxed{V = C\sqrt{mi}}$$ .... Chezy's formula.

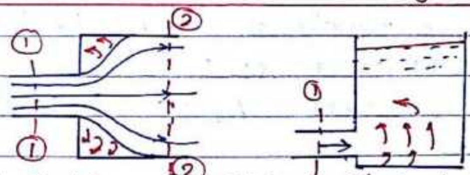
where $C = \sqrt{\frac{\rho g}{f'}}$ .. known as Chezy's constant

$i = \frac{hf}{L}$ ---- Loss of head per unit length of pipe

Minor Energy losses → / Losses due to Geometric changes.

The loss of head due to friction is the major loss & calculated from Darcy & Chezy's formula. Loss of mechanical energy due to viscosity of fluid which causes friction between the layers of fluid and between the solid surface & adjacent fluid layer. Due to friction part of mech. energy converted into intermolecular energy termed as loss of energy. Apart from friction loss of energy due to abrupt change in c/s. In long pipes these losses are negligible hence called as minor losses but considered in short pipes.

1) Losses due to sudden enlargement →



1) Sudden expansion i.e. finite enlargement ② Flow at infinite enlargement (Exit loss]

streamline diverges from wall i.e. flow separation due to abrupt change results in formation of turbulent eddies & converts mechanical energy into intermolecular energy.

Sudden expansion loss

$$= \frac{(v_1 - v_2)^2}{2g}$$

$$= \frac{v_1^2}{2g}\left[1 - \left(\frac{v_2}{v_1}\right)\right]^2$$

But $A_1 V_1 = A_2 V_2$ $\quad \frac{V_2}{V_1} = \frac{A_1}{A_2}$

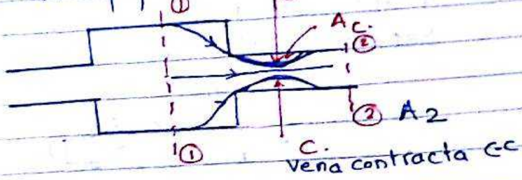
$$\boxed{h_L = \frac{v_1^2}{2g}\left[1 - \left(\frac{A_1}{A_2}\right)\right]^2} \quad -①$$

2) Exit loss → $A_2 \rightarrow \infty$ & the fluid velocities are arrested in a large reservoir the entire K.E. at the outlet of pipe is dissipated into the intermolecular energy of the reservoir through the creation of turbulent eddies hence loss is usually termed as exit loss for the pipe & equals to the velocity head at the discharge end of pipe.

when $A_2 \rightarrow \infty$ eq^n ① becomes

$$\boxed{h_L = \frac{v_1^2}{2g}}$$

3) Losses due to sudden contraction → Streamlines converges abruptly due to contraction in area. However immediately downstream of the junction of area contraction the c/s area of the stream tube becomes the minimum & less than that of the smaller pipe. This section of tube is known as Vena contracta; after which the stream wides again to fill pipe.



$$h_L = \frac{V_2^2}{2g}\left[\frac{A_2}{A_c} - 1\right]^2 \quad -②$$

But, $C_c \Rightarrow$ Coefficient of contraction $\Rightarrow \frac{A_c}{A_2}$

$$h_L = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$h_L = k\frac{V_2^2}{2g}$$

Where $k = \left[\frac{1}{C_c} - 1\right]^2$

* The value of k depends on $\frac{A_2}{A_1}$.

4) Entry loss → $A_1 \rightarrow \infty$ the $k = 0.5$

ie for $\frac{A_2}{A_1} = 0$ $\quad k = 0.5$

$$\boxed{h_L = 0.5\frac{V_2^2}{2g}} \quad \boxed{h_L = k\frac{V_2^2}{2g}}$$

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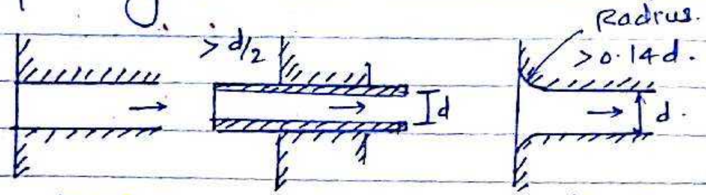
1) Entry loss due to flow of fluid from a large reservoir into a sharp edged pipe. For no potrude $k = 0.5$.

2) For potruding pipe causes greater loss of head $k = 1$

3) If the pipe inlet is well rounded the fluid can follow the boundary without separating it. $k = 0$ i.e. No entry loss.



$k = 0.5$ $k = 1$ $k = 0$

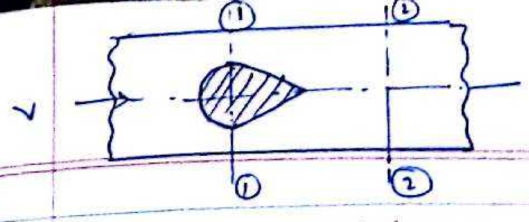
5) Loss of head due to Bend in pipe →

Due to bend flow seperation from the boundary & formation of eddies. Thus energy is lost.

$$h_b = \frac{kv^2}{2g}$$

$k \Rightarrow$ Coefficient of bend

k depends on 1) Angle of bend
2) Radius of curvature of bend
3) Diameter of pipe.



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6) Loss of head due to an obstruction in a pipe →

$$h_L = \frac{v^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2$$

where, $a \Rightarrow$ Max. area of obstruction
$A \Rightarrow$ Area of pipe
$v \Rightarrow$ velocity of liquid in pipe.

$$h_L = \frac{(v_c - v)^2}{2g}$$

$v_c \Rightarrow$ velocity at vena contracta.

$C_c \Rightarrow \frac{a_c}{A-a}$ $a_c \Rightarrow$ Area at vena contracta.

* Vena contracta is formed beyond section 1-1 after which stream of fluid widens again.

7) Loss of head in various pipe fittings →

$$h_L = \frac{kv^2}{2g}$$

$k \Rightarrow$ Coefficient of pipe fittings such as valves, couplings etc.

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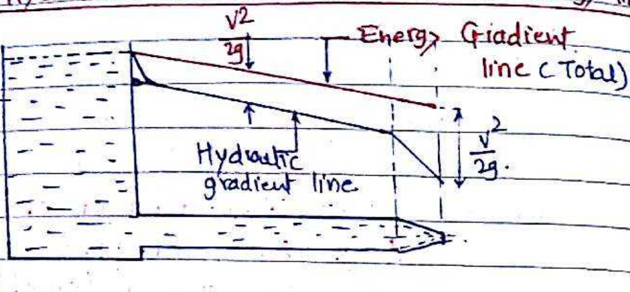
* Hydraulic Gradient line & Total energy line →



* Total or Energy gradient energy line is the contour of the total mechanical energy per unit weight or the total head $\left(\frac{P}{\rho g}+\frac{V^2}{2g}+z\right)$ at a c/s, as ordinate against the distance along the flow. (TEL)

* Hydraulic gradient line is obtained by plotting the sum of potential & pressure heads $\left(\frac{P}{\rho g}+z\right)$ as ordinate against the same abscissa (distance along the flow) i.e. by subtracting velocity head from energy gradient line. (HGL)

* TEL decreases due to frictional head loss in the pipeline & nozzle.

* HGL runs below TEL & decreases sharply in nozzle due to decrease in pressure in nozzle. It can jump also where increase in pressure & decrease in velocity.

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* HGL may rise or fall depends on sudden enlargement or contraction of the pipe at any section.

* If pump or turbine is fitted in system then TEL would show rise across the pump by an amount of head developed or abrupt fall across turbine by an an amount equal to head extracted.

* HGL & TEL meets at reservoir when velocity is negligible.

Viscous Flow

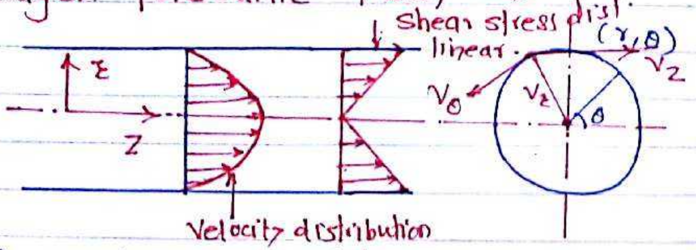
Fluid flow is governed by equation of motions like

1) Continuity equation.
2) Momentum equation [ie. Navier stokes equation] --- Viscous effects considered.
3) Energy equation.

Euler equation → It govern's ideal fluid flow where viscosity is zero ie. inviscid flow.

Bernaulli equation → Governs fluid flow which is inviscid, incompressible & steady.

* Viscous incompressible fully developed laminar flow through circular tube/pipe [Hagen Poiseuille Flow) →



Assumptions → Parabolic

1) Fully developed flow ie. $\frac{\partial v_z}{\partial z} = 0$
2) No rotational component or irrotational flow $v_\theta = 0$
3) Flow is axially symmetric
$$\frac{\partial}{\partial \theta}(\text{any variable}) = 0$$

Continuity eq^n in cylindrical co-ordinates is given by

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}\rho v_\theta + \frac{\partial}{\partial z}\rho v_z = 0$$

under assumptions this eq^n reduces to

$$\frac{\partial}{\partial r}\rho r v_r = 0$$

for incompressible flow ρ = const.

$$\frac{\partial}{\partial r} r v_r = 0$$

Above eq^n implies that $r v_r$ is not a function of r. By nonpenetration boundary cond^n at the wall implies $v_r = 0$ everywhere in the flow except at $r = 0$. which is singularity. Hence for fully developed flow only one velocity component $v_z = v_z(r)$

Let's take momentum or Navier stokes eq^n.

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right)$$
$$= -\frac{\partial p^*}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]$$

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With above assumptions Navier stoke's equation

$$0 = -\frac{\partial p^*}{\partial r}$$

In other words p^* is not a function of r. Therefore p^* is function z only

Navier stokes equation cylindrical co-ordinates →

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p^*}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

Above eq^n under assumptions

$$0 = -\frac{\partial p^*}{\partial z} + \mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)$$

$$\frac{dp^*}{dz} = \mu\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)$$

Since $p^* = f(z)$ & u is $f(r)$ only

$$\frac{dp^*}{dz} = \mu\frac{1}{r}\frac{d}{dr}\left[r\frac{dv_z}{dr}\right] = C \text{ (say)} \quad -(1)$$

$$\mu\frac{1}{r}\frac{d}{dr}\left[r\frac{dv_z}{dr}\right] = C$$

Multiply both sides $\frac{r}{\mu}$

$$\frac{d}{dr}\left[r\frac{dv_z}{dr}\right] = \frac{Cr}{\mu}$$

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Integrating w.r.t. r.

$$r\frac{dv_z}{dr} = \frac{Cr^2}{2\mu} + C_1$$

Dividing both sides by r

$$\frac{dv_z}{dr} = \frac{Cr}{2\mu} + C_1\frac{1}{r}$$

Integrating

$$v_z = \frac{Cr^2}{4\mu} + C_1 \ln r + C_2$$

To ensure finite velocity at the channel centerline (avoiding logarithmic singularit we must have $C_1 = 0$

~~Thus~~,

Also, boundary cond^n are no-slip at wal & finite velocity at the centreline. The former ($v_z = 0$ at $r = R$) results in,

$$0 = \frac{Cr^2}{4\mu} + C_1 \ln r + C_2$$

$$C_2 = -\frac{CR^2}{4\mu}$$

Hence

$$v_z = \frac{Cr^2}{4\mu} - \frac{CR^2}{4\mu}$$

$$v_z = -\frac{C}{4\mu}(R^2 - r^2)$$

from eq^n (1) we can write.

$$\boxed{v_z = -\frac{1}{4\mu}\frac{dp^*}{dz}(R^2 - r^2)}$$

Scanned by CamScanner

Taking common R^2

$$\boxed{v_z = \frac{-R^2}{4\mu}\frac{dp^*}{dz}\left(1-\frac{r^2}{R^2}\right)} \quad \text{--- (I)}$$

* Above eq^n shows that the axial velocity profile in a steady, fully developed laminar flow through a circular tube has a parabolic variation along r.
The max. velocity occures at the centre line. when $r = 0$

$$\boxed{V_{z max} = -\frac{R^2}{4\mu}\frac{dp^*}{dz}}$$

Also,

$$\boxed{\frac{V_z}{V_{zmax}} = \left(1-\frac{r^2}{R^2}\right)}$$

Volume flow rate

$$Q = \int V_z\, dA$$

Avg. velocity $\bar{V}_{z\,avg} = \frac{Q}{A} = \frac{\int_0^R V_z\, 2\pi r\, dr}{\pi R^2}$

Let $= 2\pi\int_0^R V_z r\, dr.$

$$= 2\pi\int_0^R -\frac{1}{4\mu}\frac{dp^*}{dz}(R^2-r^2)\, r\, dr.$$

$$= \frac{-2\pi}{4\mu}\frac{dp^*}{dz}\int_0^R (R^2 r - r^3)\, dr$$

$$= -\frac{2\pi}{4\mu}\frac{dp^*}{dz}\left[R^2\frac{r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= \frac{-\cancel{2}\pi}{{}_2\cancel{4}\mu}\frac{dp^*}{dz}\left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= -\frac{\pi}{2\mu}\frac{dp^*}{dz}\left[\frac{R^4}{4}\right]$$

$$\bar{V}_{z\,avg} = \frac{\frac{-\cancel{\pi}}{2\mu}\frac{dp^*}{dz}\frac{R^{\cancel{4}\,2}}{4}}{\cancel{\pi}R^2}$$

$$\boxed{\bar{V}_{z\,Avg} = \frac{-R^2}{8\mu}\frac{dp^*}{dz}} \quad \text{--- (2)}$$

* Ratio of max. velocity to avg. velocity →

$$\frac{V_{z max}}{\bar{V}_{z avg}} = \frac{\frac{-R^2}{4\mu}\cancel{\frac{dp^*}{dz}}}{\frac{-R^2}{8\mu}\cancel{\frac{dp^*}{dz}}}$$

$$\boxed{\frac{V_{z max}}{\bar{V}_{z avg}} = 2}$$

* Shear stress distribution in pipe →
Shear stress at any location

$$\tau = \tau_{rz} = \mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)$$

Since $v_r = 0$ & v_z is $f(r)$ only.

$$\tau = \frac{\mu\, dv_z}{dr}$$

$$= \mu \frac{d}{dr}\left[-\frac{1}{4\mu}\frac{dp^*}{dz}(R^2-r^2)\right]$$

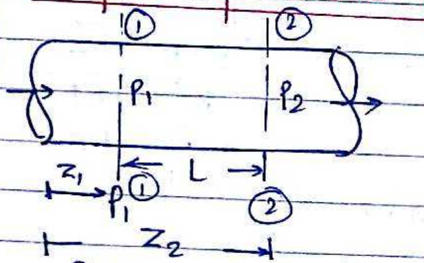
$$= \frac{-\cancel{\mu}}{4\cancel{\mu}}\frac{dp^*}{dz}\cdot\frac{d}{dr}(R^2-r^2)$$

$$= -\frac{dp^*}{\cancel{4}_2\, dz}(-\cancel{2}r)$$

$$\boxed{\tau = \frac{r}{2}\frac{dp^*}{dz}} \qquad \tau_w = \frac{R}{2}\frac{dp^*}{dz} \quad \text{— (3)}$$

* Above equation shows shear stress varies linearly with the radial distance from axis.

* Drop of pressure for a given length of pipe →



From eq^n (2)

$$\frac{dp^*}{dz} = \frac{-8\mu \bar{V}_2}{R^2}$$

Since $\frac{dp^*}{dz}$ is linear considering piezometric pressures at upstream & downstream

$$\frac{p_2^* - p_1^*}{z_2 - z_1} = \frac{-\Delta p^*}{L}$$

Since $\Delta p^* = p_1^* - p_2^*$

$$\frac{\Delta p^*}{L} = \frac{8\mu \bar{V}_2}{R^2}$$

$$\boxed{\Delta p^* = \frac{32\mu \bar{V}_2 L}{D^2}} \quad \dots R = \frac{D}{2}$$

$$Q = A \times \bar{V}_2$$

$$Q = \frac{\pi}{4}D^2 \times \bar{V}_2 \qquad \bar{V}_2 = \frac{4Q}{\pi D^2}$$

$$\boxed{\Delta p^* = \frac{128\mu Q L}{\pi D^4}} \quad \dots \text{Hagen Poiseuille equation}$$

$$h_f = \frac{\Delta p^*}{\rho g}$$

$$\boxed{h_f = \frac{128\mu Q L}{\rho g \pi D^4}}$$

$$h_f = f\frac{L}{D}\frac{\bar{V}_2^2}{2g} = \frac{32\mu \bar{V}_2 L}{\rho g D^2}$$

$$\boxed{f = \frac{64}{\frac{\rho \bar{V}_2 D}{\mu}} = \frac{64}{Re_D}} \quad \text{— (4)}$$

where f = Darcy's friction factor.

skin friction or fanning friction coefficient is defined as,

$$C_f = \frac{|\tau_\omega|}{\frac{1}{2}\rho \bar{v}_z^{\,2}}$$

Putting values of $\bar{v}_z$ & τ_w

$$C_f = \frac{\cancel{R}/2 \; \cancel{dp^*/dz}}{\frac{1}{2} \times \rho \times \left(-\frac{R^{\cancel{2}}}{8\mu}\cancel{\frac{dp^*}{dz}}\right)\bar{v}_z}$$

$$C_f = \frac{8\mu}{R\rho \bar{v}_z} = \frac{16}{\frac{\rho \bar{v}_z D}{\mu}} \qquad R = \frac{D}{2}$$

$$\boxed{C_f = \frac{16}{Re_D}} \quad - (5)$$

$$\frac{C_f}{f} = \frac{16/Re_D}{64/Re_D} = \frac{1}{4}$$

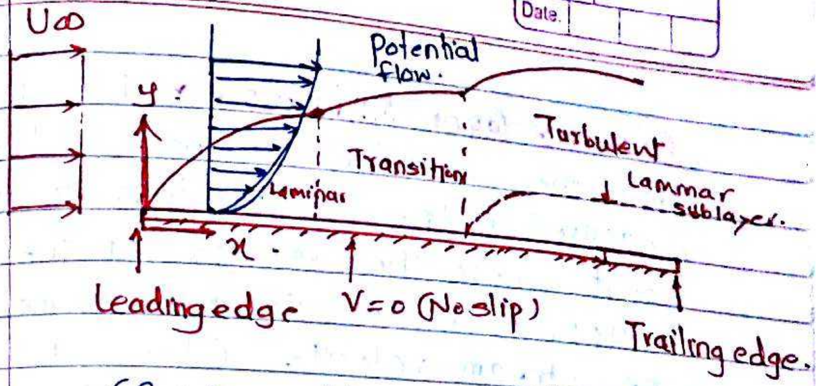
$$\boxed{f = 4C_f}$$

Note. Hence the Darcy friction factor is 4 times the fanning friction coefficient.

BOUNDARY LAYER THEORY

When a real fluid passes past a solid body or a solid wall, the fluid particles adhere to the boundary & condⁿ of no slip occures. This means velocity of fluid close to the boundary will be same as that of the boundary. If the boundary is stationary then velocity of fluid at the boundary will be zero. Further away from boundary, the velocity will be higher & as a result of this variation of velocity the velocity gradient $\frac{du}{dy}$ will exist. The velocity of ~~flu~~ boundary increases zero velocity on the stationary boundary to the free stream velocity. (U_∞). This variation of velocity from zero to free stream velocity in the direction normal to the boundary takes place in a narrow region in the vicinity of solid boundary. The narrow region of the fluid is called boundary layer.

The remaining fluid which is outside the boundary layer is constant & equal to free-stream velocity. As there is no variation in this region, the velocity gradient $\frac{du}{dy}$ becomes zero. As a result of this shear stress is zero. [Potential flow]



$$(Re_x) = \frac{U_\infty \times x}{\nu} \qquad Re = 5 \times 10^5 \therefore \text{Laminar flow}$$

$x \Rightarrow$ Distance from leading edge

$U_\infty \Rightarrow$ free stream velocity.

$\nu \Rightarrow$ kinematic viscosity of fluid.

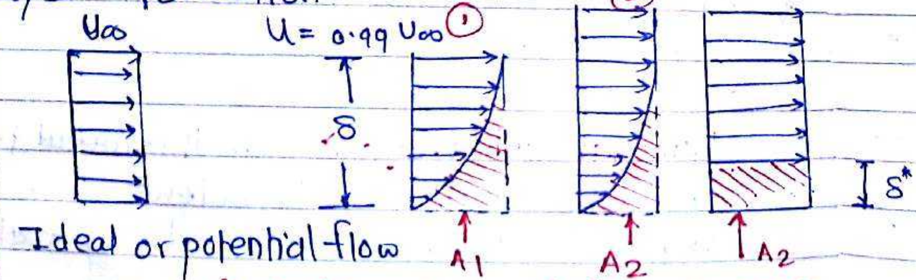
Boundary layer increases as distance increases in x direction. As $y=0$ $V=0$ No slip further increase in U due to shear resistance decreases U_∞ layer by layer

Laminar Sublayer → ~~In~~ This is ~~used~~ formed near the wall in turbulent zone. In this region velocity dist. is also parabolic but its very small thickness assume the velocity variation is linear & $\frac{du}{dy} = \text{const.}$ hence τ is also constant & equal to τ_{wall}.

$$\tau_w = \mu \left(\frac{du}{dy}\right)_{y=0} = \frac{\mu u}{y}$$

Boundary layer thickness → Distance from the boundary of the solid body measured in the y-direction to the point where the velocity of the fluid is approx. equal to 0.99 times of free stream velocity (U_∞) of the fluid. It is denoted by δ.

Displacement thickness → (δ^*) the distance measured perpendicular to the boundary of solid body, by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation.



$A_1 > A_2$ loss of flow is more hence first flow is not good. Hence displacement thickness gives the representation of loss in mass flow rate.

let mass $= \dot{m} = \rho A V$.

$$\dot{m}_{ideal} = \int_0^\delta \rho\,(dy \times 1)\,U_\infty$$

$$\dot{m}_{real} = \int_0^\delta \rho\,(dy \times 1)\,u$$

$$\dot{m}_{loss} = \int_0^\delta \rho\,dy\,(U_\infty - u)$$

Also,

$$\dot{m}_{loss} = \rho\,(\delta^* \times 1)\,U_\infty$$

where δ^* = Displacement thickness.

$$\rho\,(\delta^* \times 1)\,U_\infty = \int_0^\delta (U_\infty - u)\,\rho\,dy$$

$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy}$$

Momentum Thickness (θ) → Momentum thickness is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation. denoted by θ.

$$F = ma = m\frac{dv}{dt}$$

$$= m\left(\frac{v_2 - v_1}{\Delta t}\right)$$

$$\text{Force}_{loss} = \dot{m} v_2 - \dot{m} v_1$$

$$\int_0^\delta \rho u (U_\infty - u)\,dy = \rho\,(\theta \times 1) \times U_\infty \times U_\infty$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy.$$

Momentum Thickness $\theta = \int_0^\delta \frac{u}{U_\infty}\left[1-\frac{u}{U_\infty}\right] dy$

* θ is greater loss in force is more.

Energy thickness → distance measured perpendicular to the boundary of the solid body, by which boundary should be displaced to compensate for the reduction in kinetic energy of the flowing fluid due to boundary layer formation. denoted by δ^{**}.

$KE = \frac{1}{2} m v^2$

Absence of boundary layer

$k.E = \frac{1}{2}(\rho u b dy) U_\infty^2$

when boundary layer exist

$KE = \frac{1}{2}(\rho u b dy) u^2$

Loss in k.E $= \frac{1}{2} \rho \int_0^\delta u b [U_\infty^2 - u^2] dy$ — ①

Loss of k.E through δ^{**} of fluid with velocity U

$= \frac{1}{2} \times m \times v^2 = \frac{1}{2}(\rho \times area \times v\, dt) \times v^2$

$= \frac{1}{2} \times (\rho \times b \times \delta^{**} \times U_\infty) U_\infty^2$

$= \frac{1}{2} \rho b \delta^{**} U_\infty^3$ — ②

Eq ① = ②

$\frac{1}{2} b \rho \int_0^\delta u [U_\infty^2 - u^2] dy = \frac{1}{2} \rho b \delta^{**} U_\infty^3$

$\delta^{**} = \frac{1}{U_\infty^3} \int_0^\delta u (U_\infty^2 - u^2) dy$

$$\boxed{\delta^{**} = \int_0^\delta \frac{u}{U}\left[1 - \frac{u^2}{U_\infty^2}\right] dy}$$

Von Karman momentum integral equation →

$$\boxed{\frac{\tau_0}{\rho U_\infty^2} = \frac{\partial \theta}{\partial x}}$$

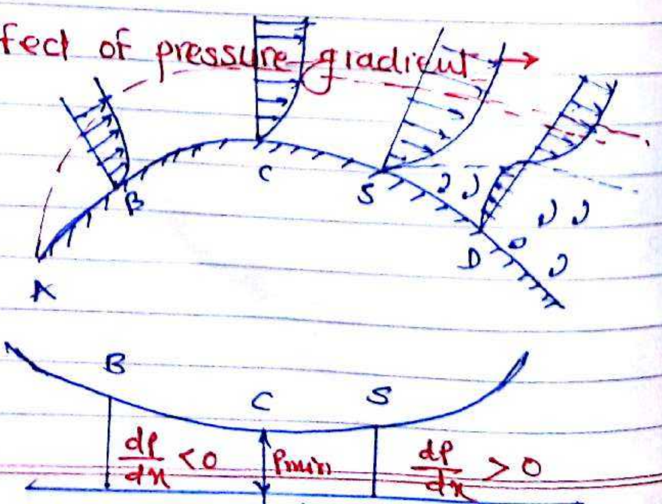
where θ = Momentum thickness

τ_0 = τ_{wall}

Separation of boundary layer →

In the boundary layer, the fluid layer adjacent to the solid surface has to do work against surface friction at the expense of its kinetic energy. This loss of the K.E is recovered from the immediate fluid layer in contact with the layer adjacent to the solid surface through momentum exchange process. Thus the velocity goes on decreasing. Along the length of the body at a certain point a stage may come when the boundary layer may not be able to keep sticking to the solid body if it can't provide K.E to overcome the resistance offered by solid body. the boundary layer separates from solid body.

Effect of pressure gradient →



In region ABC, area of flow decreases & hence pressure decreases & velocity increases i.e. flow acclerated in this region hence pressure gradient $\frac{dp}{dx}$ is negative.

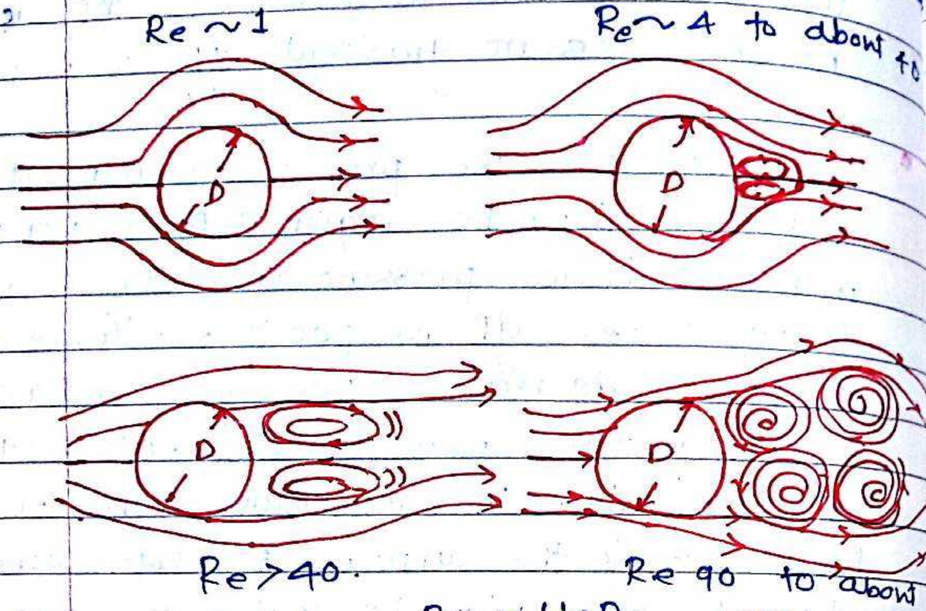
In region CSD, the pressure is min. at the point C. Along the region CSD, area of flow increases hence pressure increases & velocity decreases i.e. $\frac{dp}{dx}$ is positive. Hence K.E of layer decreases. Thus the combined effect of positive pressure gradient & surface resistance reduce the momentum of the fluid is unable to the surface. A stage comes, when the momentum is unable to overcome the surface resistance & boundary layer starts separating from the surface at the pt. S & flow is taking place in reverse direction.

Imp. Note: Thus the positive pressure gradient helps to separate the boundary layer.

* Location of seperation point → The separation point S at $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$ .... i.e. flow is on the verge of separation.

* $\left(\frac{\partial u}{\partial y}\right)_{y=0} = -ve$ .... The flow has separated

* If $\left(\frac{\partial u}{\partial x}\right)_{y=0}$ = +ve ... The flow will n[ot] separate. i.e. attached f[low]



$Re = \frac{U_\infty D}{\nu}$

Flow past over cylinder;

1) when Re ~ 1 the flow smoothly divides and reunites around the cylinder.
2) At a Re ~ 40, the flow separates in the downstream & the wake is formed by two symmetric eddies. The eddies remain steady & symmetrical but grow in size up to Re of about 40.
3) Re > 40 oscillation in the wake induces asymmetry & finally the wake starts shedding vortices into the stream, termed as onset of periodicity.
4) ~~Periodicity induces~~ Re > 90 the eddies are shed alternatively from a top & bottom of the cylinder & the regular pattern of altenatively shed clockwise & counter clk. vortices form Von Karmann vortex street. However periodicity is eventually induced in the flow field with the vortex shedding phenomenon. Vortex periodicity frequency of vortex shedding f.

Each time a vortex is shed from the cylinder, a circulation is produced and consequently an unbalanced lateral force acts on the cylinder. The shedding of vortices alternately from the top & bottom of the cylinder produces alternating lateral forces causing vibrations at the same frequency of the shedding frequency. This is the cause of singing of telephone wires in the breeze. In the aircrafts this phenomenon is known as flutter.

Shading frequency given by :

$$\boxed{\frac{fD}{U_\infty} = 0.198\left(1 - \frac{19.7}{Re}\right)}$$

The formula works for $250 < Re < 2 \times 10^5$

FM&HM

1.. Differential manometers are used to measure the difference of pressures between two points in a pipe or in two different pipes. There are two types of differential manometers.

1. U-tube upright differential manometer
2. U-tube inverted differential manometer

2.. Total Pressure and Centre of Pressure

The total pressure is defined as the force exerted by a static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface. This force is always normal to the surface. The centre of pressure is defined as the point of application of the resultant pressure on the surface.

3..

Types of fluid flow:

- Steady and Unsteady **flows**: ...
- Uniform and Non-uniform **fluid flow**: ...
- Laminar, and Turbulent **fluid flow**: ...
- Compressible and Incompressible **fluid flow**: ...
- Rotational and irrotational **Fluid flow**: ...
- One, Two and Three-dimensional **fluid Flow**:

4..L **aminar flow** or streamline flow in pipes (or tubes) occurs when a fluid flows in parallel layers, with no disruption between the layers. At low velocities, the fluid tends to flow without lateral mixing, and adjacent layers slide past one another like playing cards. There are no cross-currents perpendicular to the direction of flow, nor eddies or swirls of fluids. In laminar flow, the motion of the particles of the fluid is very orderly with all particles moving in straight lines parallel to the pipe walls. Any lateral mixing (mixing at right angles to the flow direction) occurs by the action of diffusion between layers of the liquid. Diffusion mixing can be slow however if the diameter of the pipe of tube is small then this diffusive mixing can be very significant.

Turbulent flow is a flow regime characterized by chaotic property changes. This includes rapid variation of pressure and flow velocity in space and time. In contrast to laminar flow the fluid no longer travels in layers and mixing across the tube is highly efficient. Flows at Reynolds numbers larger than 4000 are typically (but not necessarily) turbulent, while those at low Reynolds numbers below 2300 usually remain laminar. Flow in the range of Reynolds numbers 2300 to 4000 and known as transition.

5.. **Discharge (also called flow rate)**
The amount of fluid passing a section of a stream in unit time is called the discharge. If v is the mean velocity and A is the cross sectional area, the discharge Q is defined by $Q = Av$ which is known as volume flow rate. Discharge is also expressed as mass flow rate and weight flow rate.

6. **Continuity equation** states that the rate at which mass enters a system is equal to the rate at which mass leaves the system.

$$\textit{volume flow out over } A_2 = A_2 V_2 \Delta t$$

Therefore

$$\textit{mass in over } A = \rho A_1 V_1 \Delta t$$

$$\textit{mass out over } A = \rho A_2 V_2 \Delta t$$

$$\textit{So:} \quad \boxed{\rho A_1 V_1 = \rho A_2 V_2}$$

7..

7. **What is flow rate?**

Flow rate is an indication of how fast a substance move through a conduit from one place to another.

Flow rate can also be used to determine the distance a substance moves over a period of time.

Flow rate is usually expressed as Volume Flow rate and Mass Flow rate.

8. Hydraulic Coefficients include Coefficient of contraction, Coefficient of velocity, Coefficient of discharge and Coefficient of resistance. The following are the hydraulic coefficients:

1. Coefficient of contraction (Cc). It is defined as the ratio of area of jet at *vena contracta (ac) to the area of orifice (a).

The point at which the streamlines first become parallel is called vena contracta. The cross-sectional area of the jet at vena contrata is less than that of the orifice. The theoretical velocity of jet at vena contracta is given by

This expression is called Torricelli's theorem.

2. Coefficient of velocity (Cv). It is defined as the ratio of the actual velocity of the jet at vena contracta (v) to the theoretical velocity.

3. Coefficient of discharge (Cd). It is defined as the ratio of the actual discharge through the orifice (Q) to the theoretical discharge (Qth). The coefficient of discharge is equal to the product of Cc and Cv.

4. Coefficient of resistance (Cr). It is defined as the ratio of loss of head in the orifice to the head of water available at the exit of the orifice.

9. The proportion of real speed of the stream, at vena-contract a, to the hypothetical speed is known as the coefficient of speed.
Write the equation for coefficient of velocity.Cv

10. The **discharge coefficient** (also known as **coefficient of discharge** or **efflux coefficient**) is the ratio of the actual discharge to the theoretical discharge, i.e., the ratio of the mass flow rate at the discharge end of the nozzle to that of an ideal nozzle which expands an identical working fluid from the same initial conditions to the same exit pressures.

11. A **notch** refers to a deliberately introduced v-shaped, u-shaped or circular defect in a planar material. In structural components, a notch causes a stress concentration which can result in the initiation and growth of fatigue cracks. Notches are used in materials characterisation to determine fracture mechanics related properties such as fracture toughness and rates of fatigue crack growth.

Notches are commonly used in material impact tests where a morphological crack of a controlled origin is necessary to achieve standardised characterisation of fracture resistance of the material.

12. A **weir** is a barrier across the width of a river that alters the flow characteristics of water and usually results in a change in the height of the river level. There are many designs of weir, but commonly water flows freely over the top of the weir crest before cascading down to a lower level.

13.. A **piezometer** is either a device used to measure liquid pressure in a system by measuring the height to which a column of the liquid rises against gravity, or a device which measures the pressure (more precisely, the piezometric head) of groundwater at a specific point. A piezometer is designed to measure static pressures, and thus differs from a pitot tube by not being pointed into the fluid flow.

14.. A **triangular notch** gives much more accurate results in low discharge conditions, as compared to the conventional **rectangular notch**. Also, only one reading (the head) is required to calculate the discharge rate, making calculations much easier. However, it cannot handle

large volumes of flow rate accurately.

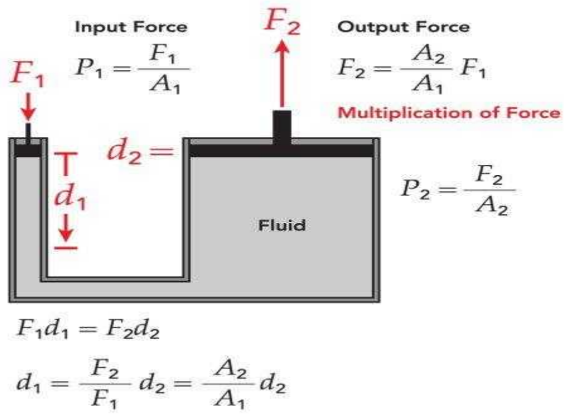
15..

1. . Simple manometer
   A. Piezometer
   B. U-tube manometer
2. Differential manometer
   A. U-Tube differential manometer
   B. Inverted U-Tube differential manometer
3. Micro manometer

LONG

1..Pascal's Law Derivation

Consider an arbitrary right-angled prismatic triangle in the liquid of density rho. Since the prismatic element is very small, every point is considered to be at the same depth from the liquid surface. The effect of gravity is also same at all these points.



Let ad, bd, and cd be the area of the faces ABFE, ABDC, and CDFE respectively.

Let P_1, P_2, and P_3 be the pressure on the faces ABFE, ABDC, and CDFE.

Pressure exerts force which is normal to the surface. Let P_1 exert force F_1 on the surface ABFE, P_2 exert force F_2 on the surface ABDC, and P_3 exert force F_3 on the surface CDFE.

Therefore, Force F_1, F_2, and F_3 is given as:
$F_1 = P_1 \times$ area of ABFE $= P_1$ ad
$F_2 = P_2 \times$ area of ABDC $= P_2$ bd
$F_3 = P_3 \times$ area of CDFE $= P_3$ cd

Also, sinθ=ba cosθ=ca
The net force on the prism will be zero since the prism is in equilibrium.

$F_1 \sin\theta = F_2$
$F_1 \cos\theta = F_3$

P_1 ad ba = P_2 bd (equ 1)
P_1 ad ca = P_3 cd (equ 2)
From 1 and 2
$P_1 = P_2$ and $P_1 = P_3$

$\therefore P_1 = P_2 = P_3$

2...Specific volume is defined as the number of cubic meters occupied by onekilogram of matter. It is the ratio of a material's volume to its mass, which is the proportional to density. Specific volume may be calculated or measured for any state of matter, but it is most often used in calculations involving gases.

The standard unit for specific volume is cubic meters per kilogram (m^3/kg), although it may be expressed in terms of milliliters per gram (mL/g) or cubic feet per pound (ft^3/lb).

Specific Volume Formulas

There are three common formulas used to calculate specific volume (ν):

1. **$\nu = V / m$** where V is volume and m is mass
2. **$\nu = 1 / \rho = \rho^{-1}$** where ρ is density
3. **$\nu = RT / PM = RT / P$** where R is the ideal gas constant, T is temperature, P is pressure, and M is the molarity

Specific Volume and Specific Gravity

If the specific volumes of two substances are known, this information may be used to calculate and compare their densities. Comparing density yields specific gravity values. One application of specific gravity is to predict whether a substance will float or sink when placed on another substance.

Specific Gravity

Finally, the specific gravity is used to compare the density of a fluid to the density of water. This is done by taking the ratio the density of the fluid in relation to water. The resulting ratio will be unit less. However, even though it is unit less, since density isn't unit less you will need to make sure you are using the same system of units for both fluid before finding the specific gravity.

3…What is Surface Tension?

Surface tension is a phenomenon in which the surface of a liquid, where the liquid is in contact with the gas, acts like a thin elastic sheet. The term surface tension is used only when the liquid is in contact with a gas (ex: when opened to the normal atmosphere). The term "interface tension" is used for the layer between two liquid.

Attractions between different chemical species cause the liquid molecules to unite together. The liquid molecules in the surface of the liquid are attracted by the molecules in the middle of the liquid. This is a type of cohesion. But the attraction between liquid molecules and air molecules (or the adhesive forces) are negligible. Therefore, this surface layer of liquid molecules acts as an elastic membrane. The surface layer of liquid molecules is under tension because there are no enough attraction forces to balance the cohesive forces act on them, thus this condition is called surface tension.

Surface tension is a phenomenon in which the surface of a liquid, where the liquid is in contact with the gas, acts like a thin elastic sheet.

Surface tension is the force on the surface of a liquid exposed to air.

Surface tension is measured as the force applied to a certain length of the liquid given by the unit N/m (Newton per meter).

What is Capillary Action?

Capillary action is the ability of a liquid to flow in narrow spaces without the assistance of, or in opposition to, external forces like gravity. It can be observed as liquid drawing through a capillary tube in the upward direction.

The capillary action occurs because of the intermolecular forces between liquid molecules and the surface of the capillary tube. Therefore, it occurs due to adhesion forces. When the diameter of the tube is sufficiently small, the liquid rises through the tube due to both adhesive and cohesive forces. The cohesive forces (attraction forces between similar molecules) cause the molecules to be drawn upward.

When a capillary tube is placed in a liquid, a meniscus is formed at the edge of the tube. Then, due to adhesion forces between liquid molecules and the wall of the tube, the liquid is pulled up until the gravitational force act on that amount of liquid is enough to overcome the adhesive force. Then the liquid molecules are pulled up due to cohesion.

Capillary action is the ability of a liquid to flow in narrow spaces without the assistance of, or even in opposition to, external forces like gravity.

Capillary action is the flow of a liquid against an external force without any assistance.

Capillary action is measured as the height of liquid column that is drawn upward, against the gravity given by the unit m (meter

4…

Vapor pressure (or vapour pressure in British English; see spelling differences) or **equilibrium vapor pressure** is defined as the pressure exerted by a vapor in thermodynamic equilibrium with its condensed phases (solid or liquid) at a given temperature in a closed system. The equilibrium vapor pressure is an indication of a liquid's evaporation rate. It relates to the tendency of particles to escape from the liquid (or a solid). A substance with a high vapor pressure at normal temperatures is often referred to as *volatile*. The pressure exhibited by vapor present above a liquid surface is known as vapor pressure. As the temperature of a liquid increases, the kinetic energy of its molecules also increases. As the kinetic energy of the molecules increases, the number of molecules transitioning into a vapor also increases, thereby increasing the vapor pressure.

The vapor pressure of any substance increases non-linearly with temperature according to the Clausius–Clapeyron relation. The atmospheric pressure boiling point of a liquid (also known as the normal boiling point) is the temperature at which the vapor pressure equals the ambient atmospheric pressure. With any incremental increase in that temperature, the vapor pressure becomes sufficient to overcome atmospheric pressure and lift the liquid to form vapor bubbles inside the bulk of the substance. Bubble formation deeper in the liquid requires a higher temperature due to the higher fluid pressure, because fluid pressure increases above the atmospheric pressure as the depth increases. More important at shallow depths is the higher temperature required to start bubble formation. The surface tension of the bubble wall leads to an overpressure in the very small, initial bubbles.

The vapor pressure that a single component in a mixture contributes to the total pressure in the system is called partial pressure. For example, air at sea level, and saturated with water vapor at 20 °C, has partial pressures of about 2.3 kPa of water, 78 kPa of nitrogen, 21 kPa of oxygen and 0.9 kPa of argon, totaling 102.2 kPa, making the basis for standard atmospheric pressure.

CAVITATION .....

Cavitation is a phenomenon in which rapid changes of pressure in a liquid lead to the formation of small vapor-filled cavities in places where the pressure is relatively low.

When subjected to higher pressure, these cavities, called "bubbles" or "voids", collapse and can generate shock wave that is strong very close to the bubble, but rapidly weakens as it propagates away from the bubble.

Cavitation is a significant cause of wear in some engineering contexts. Collapsing voids that implode near to a metal surface cause cyclic stress through repeated implosion. This results in surface fatigue of the metal causing a type of wear also called "cavitation". The most common examples of this kind of wear are to pump impellers, and bends where a sudden change in the direction of liquid occurs. Cavitation is usually divided into two classes of behavior: inertial (or transient) cavitation and non-inertial cavitation.

The process in which a void or bubble in a liquid rapidly collapses, producing a shock wave, is called inertial cavitation. Inertial cavitation occurs in nature in the strikes of mantis shrimps and pistol shrimps, as well as in the vascular tissues of plants. In man-made objects, it can occur in control valves, pumps, propellers and impellers.

Non-inertial cavitation is the process in which a bubble in a fluid is forced to oscillate in size or shape due to some form of energy input, such as an acoustic field. Such cavitation is often employed in ultrasonic cleaning baths and can also be observed in pumps, propellers, etc.

Since the shock waves formed by collapse of the voids are strong enough to cause significant damage to parts, cavitation is typically an undesirable phenomenon in machinery (although desirable if intentionally used, for example, to sterilize contaminated surgical instruments, break down pollutants in water purification systems, emulsify tissue for cataract surgery or kidney stone lithotripsy, or homogenize fluids). It is very often specifically avoided in the design of machines such as turbines or propellers, and eliminating cavitation is a major field in the study of fluid dynamics. However, it is sometimes useful and does not cause damage when the bubbles collapse away from machinery, such as in supercavitation.